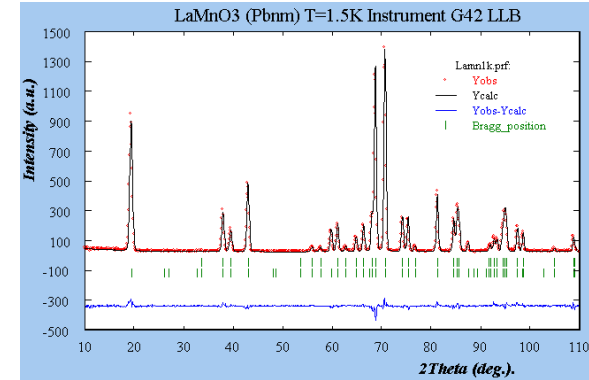


Winter school
4 – 10 February 2018,
Banyuls-sur-Mer, France



CRYSTALLOGRAPHIC and MAGNETIC STRUCTURES from NEUTRON DIFFRACTION: the POWER of SYMMETRIES (Lecture I)

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Global outline (*Lectures I, II, and III*)

I- Crystallographic structures

Point Group Symmetry: *from the elementary point symmetries to the 32 point groups*

Translation Symmetry: *lattice, motif, unit cell, directions and net planes, twins*

Space group symmetry: *glide planes and screw axes, the 230 space groups, the ITC*

II- Magnetic structures

Description in terms of propagation vector: *the various orderings, examples*

Description in terms of symmetry:

Magnetic point groups: *time reversal, the 122 magnetic point groups*

Magnetic lattices: *translations and anti-translations, the 36 magnetic lattices*

Magnetic space groups = Shubnikov groups

III- Determination of nucl. and mag. structures from neutron diffraction

Nuclear and magnetic neutron diffraction: *structure factors, extinction rules*

Examples in powder neutron diffraction

Examples in single-crystal neutron diffraction

Crystallography: *introduction*

The word **crystal** comes from Greek (*krustallas*) and means “solidified by the cold”.

Crystallography = science of crystals

→ external shape, internal structure, crystal growth, and physical properties.

Objective: determine the **atomic positions in the unit cell**.

Mean: **diffraction techniques** (X-rays, neutrons, electrons)

Curie's principle : The symmetry of a cause is always preserved in its effects



Existence or not of some phenomena, symmetries of the possible ones

Examples: existence or not of ferroelectricity
relations between the various components of the stress tensor
...

Crystallography: *introduction*

Crystallography → Link between structure and physical properties

1- Translation symmetry

Periodicity of the physical properties: **Solid state physics**

- *Phonons, magnons, ...*
- *Diffraction*

2- Point (group) symmetry

Anisotropy of the physical properties: **macroscopic physics**

→ reflects the point symmetry of crystals

- *External shape of crystals (natural faces)*
- *Electric conductivity, optical, mechanical, magnetic, properties*

To describe crystals: geometrical aspects,
symmetries, atomic positions, ...

Direct space

To determine the crystal
structure: diffraction

Reciprocal space

1. Point group symmetry

Elementary point symmetry operations

Crystallographic point groups: definition, international notation

Examples of point groups

The 32 crystallographic point groups and 11 Laue classes

2. Translation symmetry

Lattice and motif, Unit cell

The orientation symmetries of lattices:

the 6 conventional cells, 7 crystal systems and 14 Bravais lattices

Lattice directions and net planes

3. Space group symmetry

Glide planes and screw axes

The 230 space groups

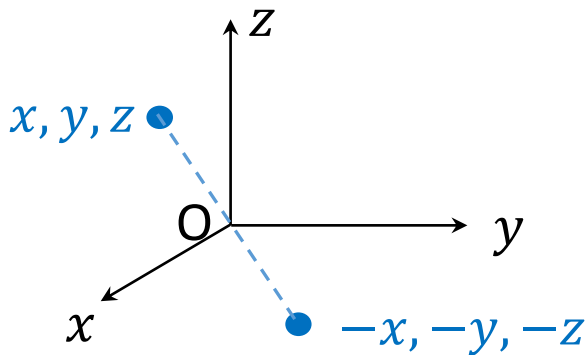
The International Tables for Crystallography

1. Point Group Symmetry: *Elementary point symmetries*

At the macroscopic & atomic (to within a translation) scales, \exists point symmetries, named **point symmetries**, that keep at least one point fixed, the origin.

Inversion (through a point)

→ *centrosymmetric crystal*

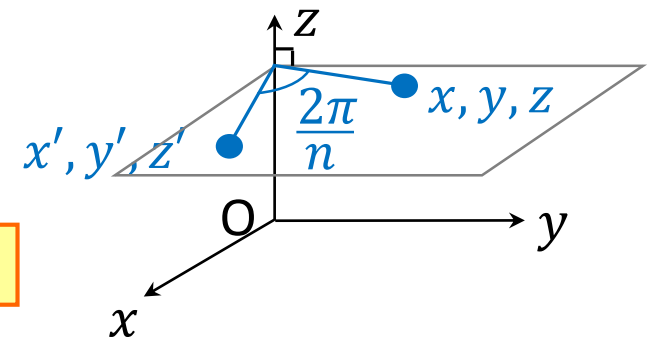


$\bar{1}$

$$\alpha(\bar{1}) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \text{Det} = -1$$

Rotation (around an axis)

Rotation of order n = rotation by $\frac{2\pi}{n}$



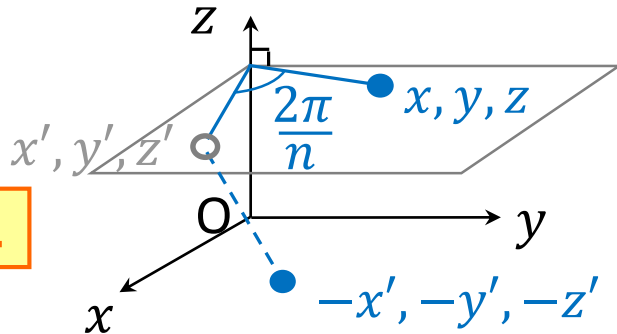
1, 2, 3, ...

$$\alpha(n) = \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{Det} = +1$$

1. Point Group Symmetry: Elementary point symmetries

Rotoinversion

(around an axis and through a point)

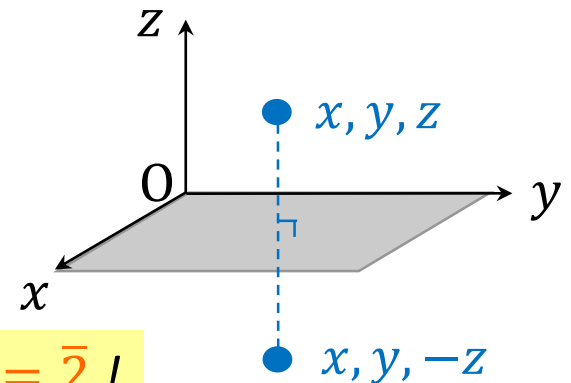


$\bar{1}, \bar{2}, \bar{3}, \dots$

$$\alpha(n) = \begin{pmatrix} -\cos \phi & \sin \phi & 0 \\ -\sin \phi & -\cos \phi & 0 \\ 0 & 0 & -1 \end{pmatrix} \text{Det} = -1$$

Reflection

(through a mirror plane)



m

NB: $m = \bar{2}$!

$$\alpha(m) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \text{Det} = -1$$

Rotations compatible with the translation symmetry = those of orders 1, 2, 3, 4, 6

→ 10 elementary operations: point groups $1, 2, 3, 4, 6, \bar{1}, \bar{2} = m, \bar{3}, \bar{4}, \bar{6}$

Proper (Det = 1) Improper (Det = -1)

1. Point Group Symmetry: *Definition of a group*

The point symmetry operations form a group

A **group** (G, \times) of order n is a set of distinct elements g_1, g_2, \dots, g_n equipped with an operation (**group multiplication** \times) that combines any two elements to form a third element and that satisfies the four axioms:

Closure: $g_i \times g_j \in G$

Identity: $\exists! e$ such that $g \times e = e \times g = g$ $\rightarrow 1$ (does nothing)

Invertibility: each element g has a unique inverse g^{-1} such that: $g \times g^{-1} = g^{-1} \times g = e$
 inverse of n : $-n$ (rotate in the other way)

Associativity: $(g_i \times g_j) \times g_k = g_i \times (g_j \times g_k)$

For point symmetry operations:

$\times \leftrightarrow$ apply successively 2 symmetry operations

1. Point Group Symmetry: *How to obtain and name all point groups?*

How to obtain all crystallographic point groups (= crystal classes) ?

Combine the 10 elementary symmetry operations, with the following constraints:

- all symmetry elements go through a common point,
- compatibility with the translation symmetry

⇒ constraints between the orientations of the various symmetry axes / planes

Notation of the point groups – *International (Hermann-Mauguin) symbol*

Symmetry operations along 1, 2 or 3 directions (**primary**, **secondary**, **tertiary**), ordered with decreasing or equal degree of symmetry
(*except for 2 cubic point groups*)

Examples :

$4/m$

$\frac{4}{m} \frac{2}{m} \frac{2}{m}$ (= $4/mmm$)

The direction of a rotation is given by its axis

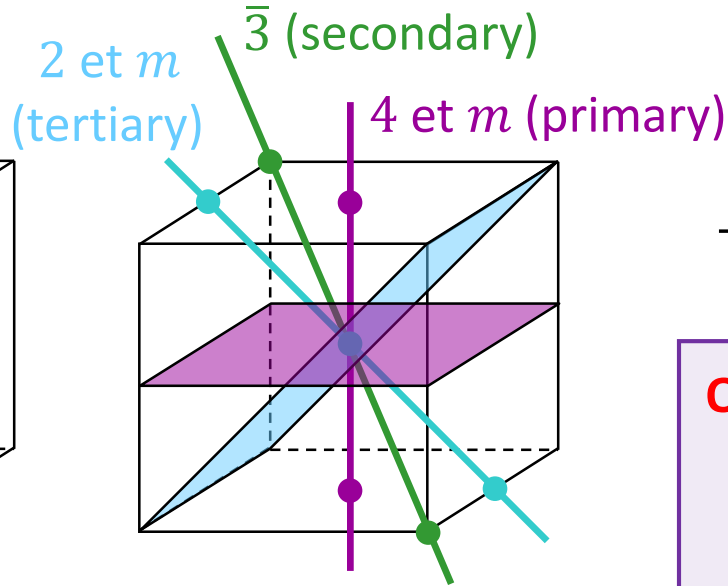
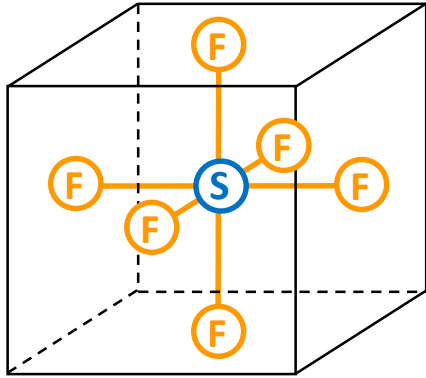
The direction of a mirror is given by its normal

' n/m ' = axis n and normal to mirror m along same direction
(*i.e.* plane of the mirror \perp to axis n)

There exists another notation: *Schoenflies symbol* → widely used in spectroscopy

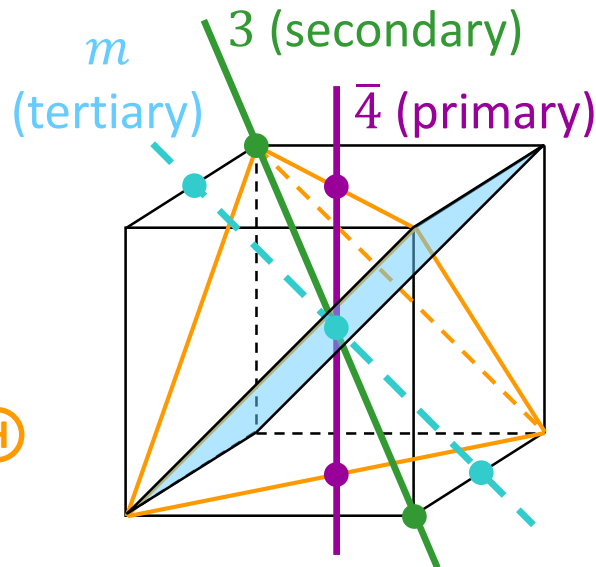
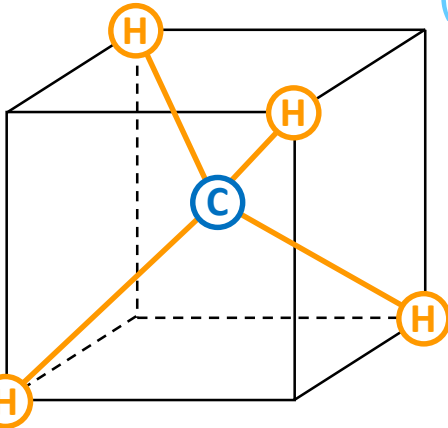
1. Point Group Symmetry: Elementary point symmetries

SF₆ molecule

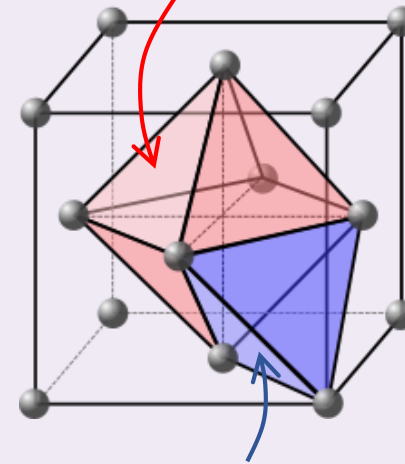


→ Point group: $\begin{matrix} 4 & 2 \\ - & \bar{3} \\ m & m \end{matrix}$ (= $m\bar{3}m$)

CH₄ molecule



Octahedral site : $m\bar{3}m$ symmetry



Tetrahedral site : $\bar{4}3m$ symmetry

→ Point group: $\bar{4}3m$

1. Point Group Symmetry: *Elementary point symmetries*

Order of the point symmetry along the:			Point groups (short symbols) and Laue classes
primary direction	secondary direction	tertiary direction	
—	—	—	1, $\bar{1}$
2	—	—	2, m , $2/m$
2	2	2	222, $2mm$, mmm
3	—	—	3, $\bar{3}$
3	2	—	32, $3m$, $\bar{3}m$
4	—	—	4, $\bar{4}$, $4/m$
4	2	2	422, $4mm$, $\bar{4}2m$, $4/mmm$
6	—	—	6, $\bar{6}$, $6/m$
6	2	2	622, $6mm$, $\bar{6}2m$, $6/mmm$
2	3	—	23, $m\bar{3}$
4	3	2	432, $\bar{4}3m$, $m\bar{3}m$

1. Point Group Symmetry: *Elementary point symmetries*

- Example: **dielectric properties**

They can only be found for particular crystal symmetries

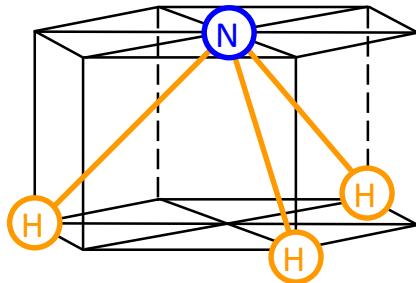
Piezoelectricity → point groups that **do not possess inversion**

Ferroelectricity and **pyroelectricity**

- **piezoelectric point groups** (i.e. non centrosymmetric)
with a unique polar axis ($\vec{p} \parallel n$ -axis and contained in the plane of the mirror):

1, 2, *m*, 2*mm*, 3, 3*m*, 4, 4*mm*, 6, 6*mm*
polar groups

NH₃



Point group: 3*m*

→ ∃ dipolar moment ($p = 1.46$ Debye)

2. Translation Symmetry: *Lattice and motif*

At the atomic scale, \exists translation vectors \vec{T} that put the crystallographic structure in coincidence with itself.

$$\vec{T} = u\vec{a} + v\vec{b} + w\vec{c} \text{ with } u, v, w \text{ integers} \quad (\text{positive or negative})$$

\vec{a} , \vec{b} , and \vec{c} are called the **basis vectors**

(non-coplanar elementary translation vectors defining a right-handed system).

The volume they define is called the **unit cell**.

Crystal = Lattice + Motif



The set of extremities of the \vec{T} vectors define an abstract network of points (= nodes): the **lattice**.

At each lattice node, one associates a group of atoms: the **motif**.

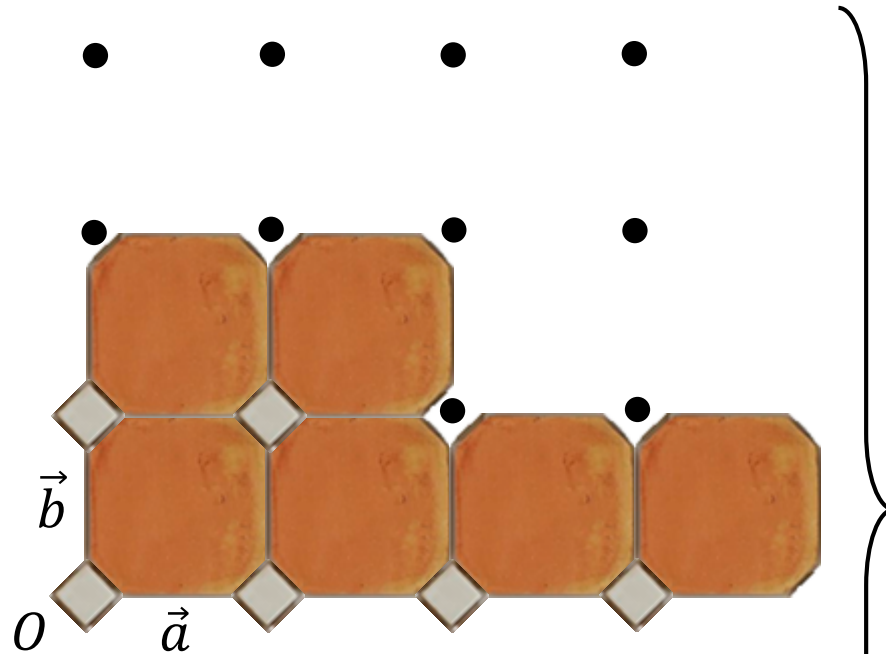
The knowledge of the lattice (basis vectors \vec{a} , \vec{b} , \vec{c}) and of the motif (nature and positions x, y, z of the atoms in the cell) completely characterizes the crystalline structure.

$$\text{N.B. : } \vec{r} = x\vec{a} + y\vec{b} + z\vec{c} \quad (|x|, |y|, |z| < 1)$$

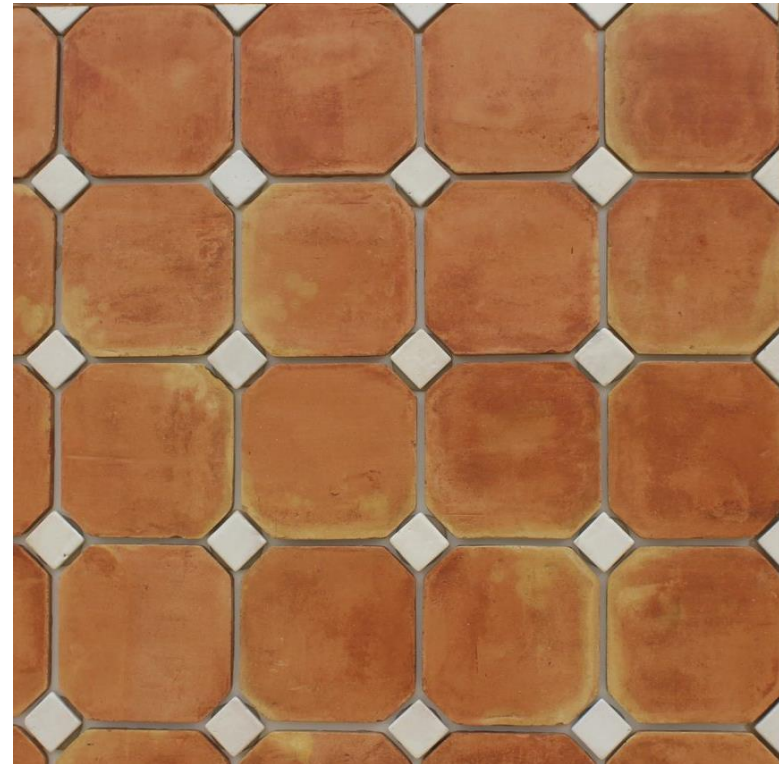
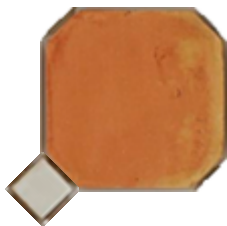
2. Translation Symmetry: *Lattice and motif*

Example 1 : terracotta floor tiles (2D)

Lattice



Motif

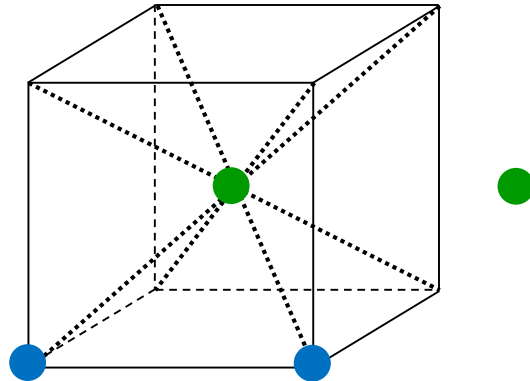


2. Translation Symmetry: *Lattice and motif*

Example 2 : CsCl single-crystal (3D)

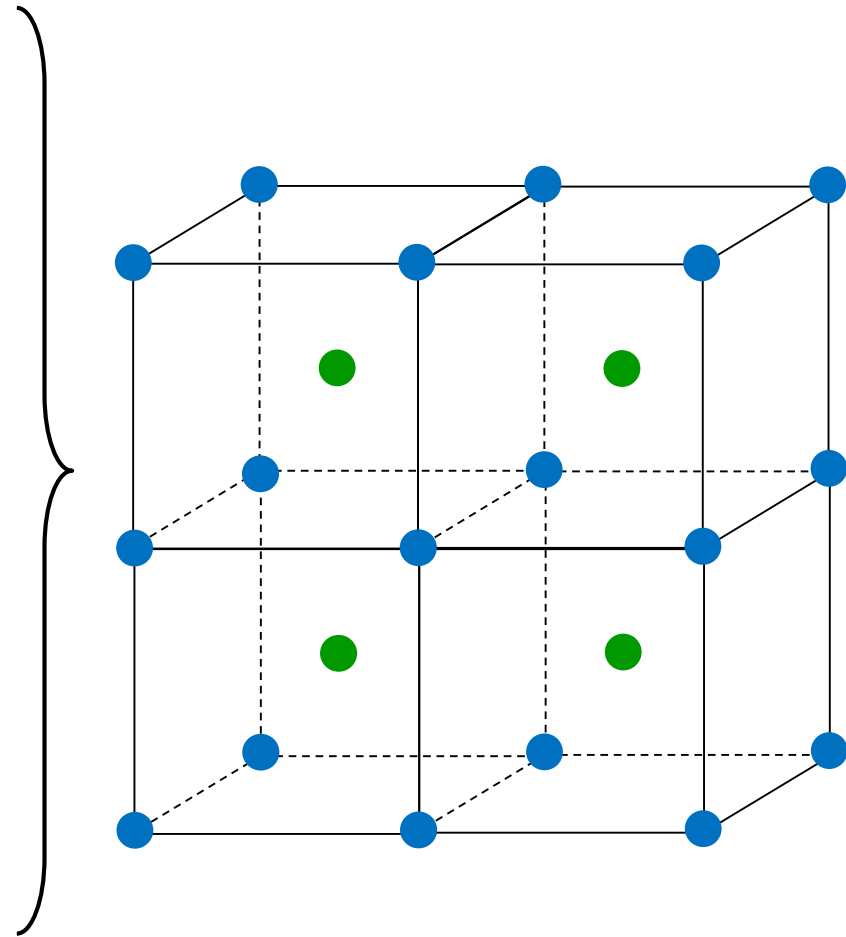
Unit cell:

cubic
primitive



Motif:

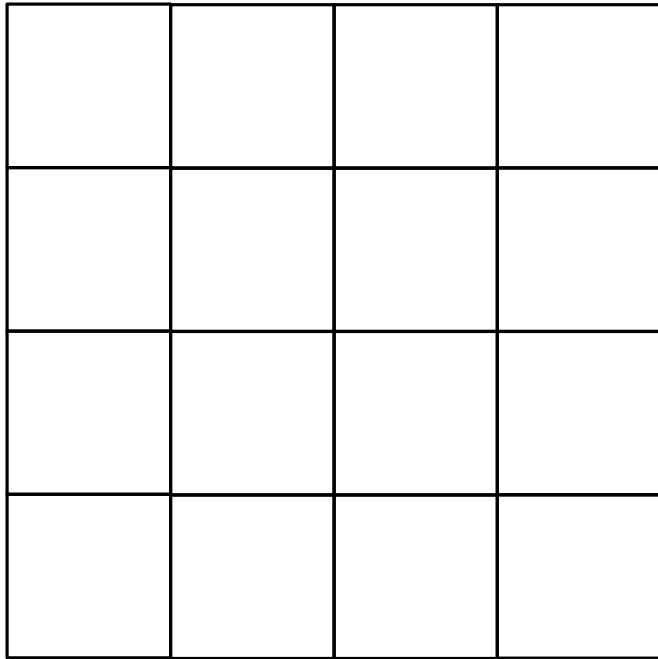
Cs⁺ on the corner
Cl⁻ at the center



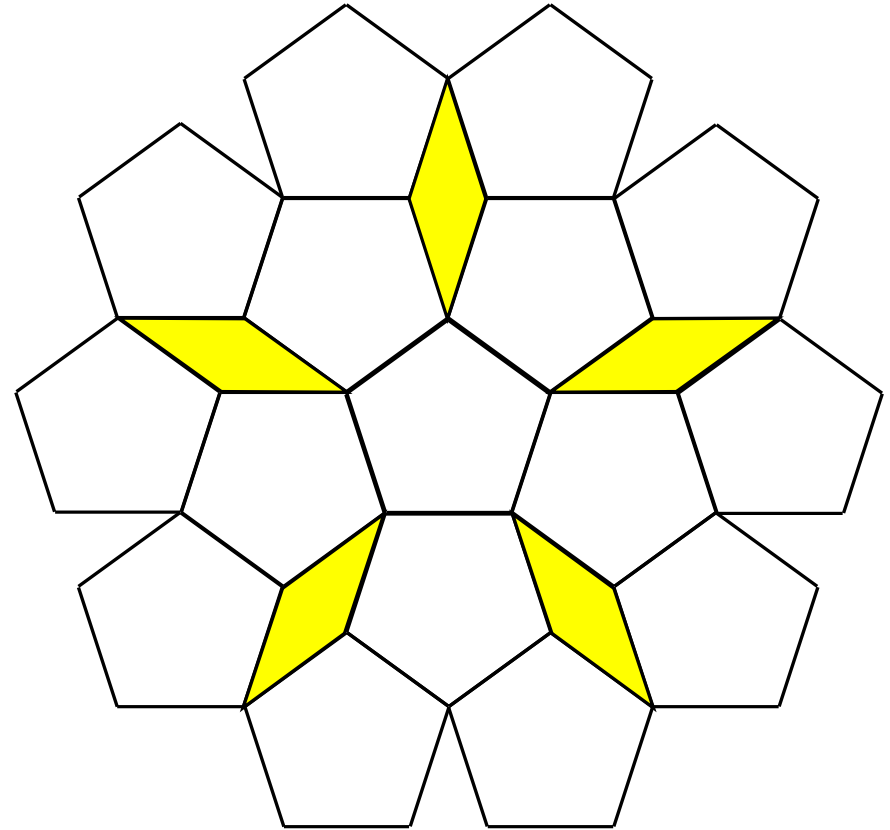
2. Translation Symmetry: *Unit cell*

The **unit cell** allows to pave the space with no empty space nor overlap, by applying the lattice translations.

Examples at 2D:

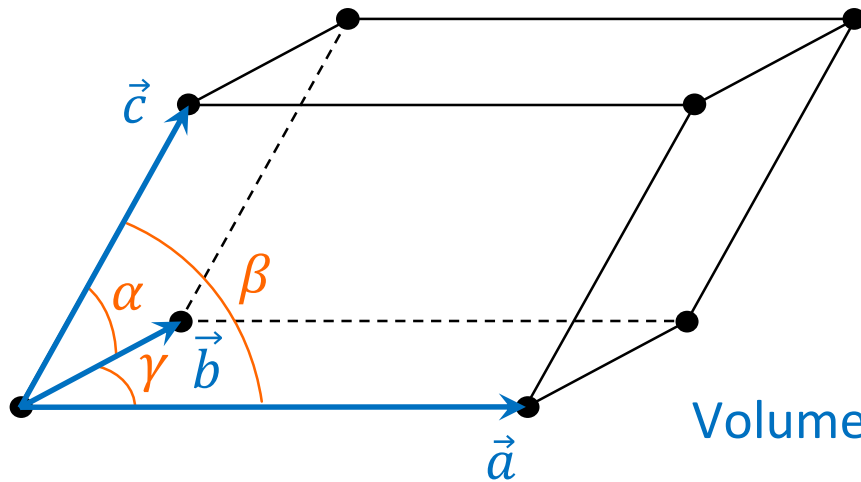


Rotation of order 4 : compatible with translation symmetry.



Rotation of order 5 : not compatible with translation symmetry → *quasicrystals*

2. Translation Symmetry: *Unit cell*



Lattice parameters:

Lengths	Angles
a	$\alpha = \widehat{(\vec{b}, \vec{c})}$
b	$\beta = \widehat{(\vec{c}, \vec{a})}$
c	$\gamma = \widehat{(\vec{a}, \vec{b})}$

Volume of the unit cell:

$$V = (\vec{a}, \vec{b}, \vec{c}) = (\vec{a} \wedge \vec{b}) \cdot \vec{c}$$

- **Multiplicity m** of a unit cell: Number of lattice nodes (and thus of motifs) per unit cell

How to count the number of lattice nodes per unit cell?

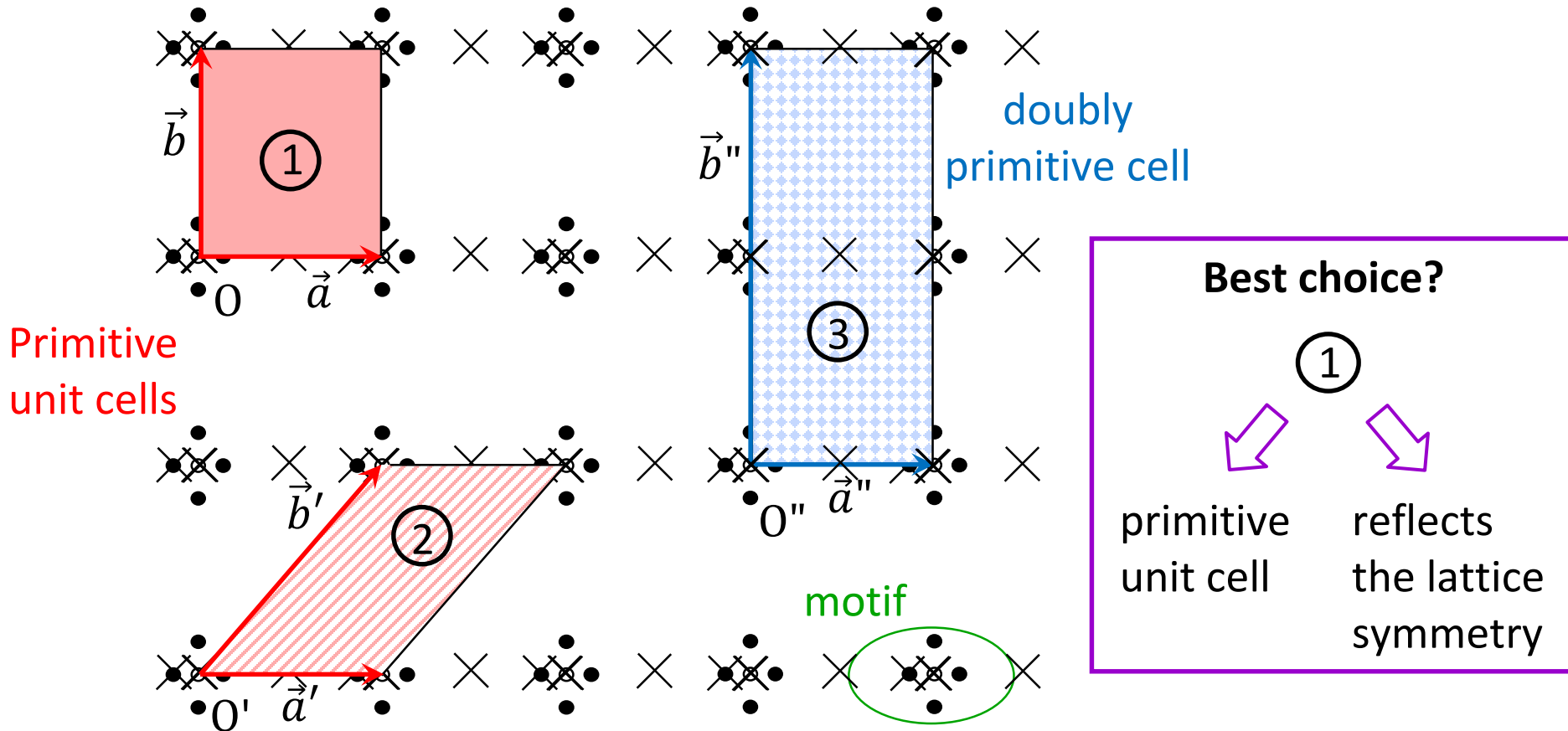
→ each lattice node counts for $1/n$, with n = number of unit cells to which it belongs

- **Primitive unit cell:** $m = 1$

For a given lattice, all primitive unit cells have the same volume V

- **Centered unit cell:** $m = 2, 3$ or 4 (**doubly, triply ... primitive**) → Volume : $V_m = m V$
 → **used only when more symmetrical than any primitive cell of the lattice**

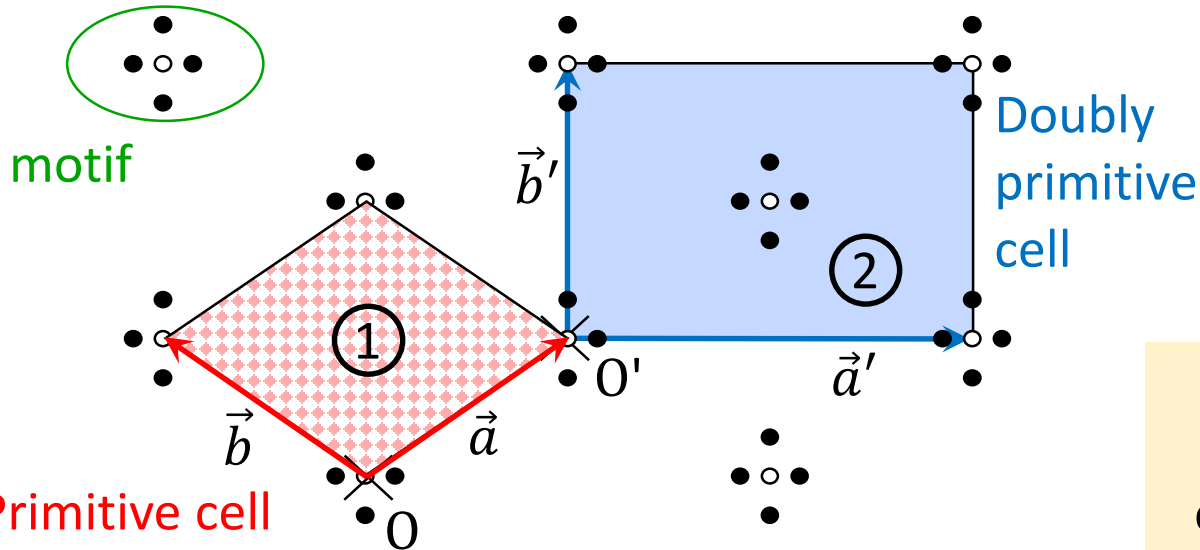
2. Translation Symmetry: *Unit cell*



Primitive cells: 4 lattice nodes (on corners) \in 4 cells $\rightarrow m = 4 \times 1/4 = 1$

Doubly primitive cell: 4 nodes (on corners) \in 4 cells $\rightarrow 4 \times 1/4 = 1$
 + 2 nodes (on edges) \in 2 cells $\rightarrow 2 \times 1/2 = 1$ } $m = 2$

2. Translation Symmetry: *Unit cell*



Cell ① is primitive but does not reflect the \perp^{ty}

➡ **Best choice:** ②

Conventional unit cell
(basis vectors \parallel directions of symmetry of the lattice)

N.B.: For a **primitive cell**, the translation vectors \vec{T} are defined by:

$$\vec{T} = u\vec{a} + v\vec{b} + w\vec{c} \text{ with } u, v, w \text{ integers.}$$

For a **non primitive cell** of multiplicity m , one must add $(m - 1)$ translation vectors such as: $\vec{T} = u'\vec{a} + v'\vec{b} + w'\vec{c}$ with u', v', w' integers or fractionals

Ex.: For unit cell ② ($m = 2$):

$$\begin{cases} \vec{T}_1 = u\vec{a}' + v\vec{b}' \\ \vec{T}_2 = \vec{T}_1 + \frac{1}{2}(\vec{a}' + \vec{b}') = \left(u + \frac{1}{2}\right)\vec{a}' + \left(v + \frac{1}{2}\right)\vec{b}' \end{cases}$$

half integers

2. Translation Symmetry: *The 6 conventional cells and 7 crystal systems*

Translation and orientation (point) symmetries:



The crystals can be classified into **6 conventional cells** and **7 crystal systems** each of them having a characteristic orientation symmetry

Number of parameters

The 6 conventional cells are, by increasing degree of symmetry:

a	triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma$
m	monoclinic	$a \neq b \neq c$	$\alpha = \gamma = 90^\circ, \beta > 90^\circ$
o	orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90^\circ$
t	tetragonal or quadratic	$a = b \neq c$	$\alpha = \beta = \gamma = 90^\circ$
h	hexagonal **	$a = b \neq c$	$\alpha = \beta = 90^\circ, \gamma = 120^\circ *$
c	cubic	$a = b = c$	$\alpha = \beta = \gamma = 90^\circ$

6

4

3

2

2

1

* $\gamma = 120^\circ$ and not 60° (for the hexagonal reciprocal lattice: $\gamma^* = 60^\circ$)

** The hexagonal conventional cell splits in two **crystal systems**:

trigonal (axis 3) and **hexagonal** (axis 6); the 5 other ones are the same.

2. Translation Symmetry: *Crystal system vs point group*

Crystal system	Point groups and Laue classes	Primary direction	Secondary direction	Tertiary direction
triclinic	1, $\bar{1}$	—	—	—
monoclinic	2, m , $2/m$	\vec{b} (ou \vec{c})	—	—
orthorhombic	222, $2mm$, mmm	\vec{a}	\vec{b}	\vec{c}

2. Translation Symmetry: Crystal system vs point group

Crystal system	Point groups and Laue classes	Primary direction	Secondary direction	Tertiary direction
triclinic	1, $\bar{1}$	—	—	—
monoclinic	2, m , $2/m$	\vec{b} (ou \vec{c})	—	—
orthorhombic	222, $2mm$, mmm	\vec{a}	\vec{b}	\vec{c}
trigonal	3, $\bar{3}$ 32, $3m$, $\bar{3}m$	\vec{c}	$\vec{a}, \vec{b}, -\vec{a}-\vec{b}$	—
tetragonal or quadratic	4, $\bar{4}$, $4/m$ 422, $4mm$, $\bar{4}2m$, $4/mmm$	\vec{c}	\vec{a}, \vec{b}	$\vec{a}+\vec{b}, \vec{a}-\vec{b}$
hexagonal	6, $\bar{6}$, $6/m$ 622, $6mm$, $\bar{6}2m$, $6/mmm$	\vec{c}	$\vec{a}, \vec{b}, -\vec{a}-\vec{b}$	$2\vec{a}+\vec{b}, \dots$

2. Translation Symmetry: *Crystal system vs point group*

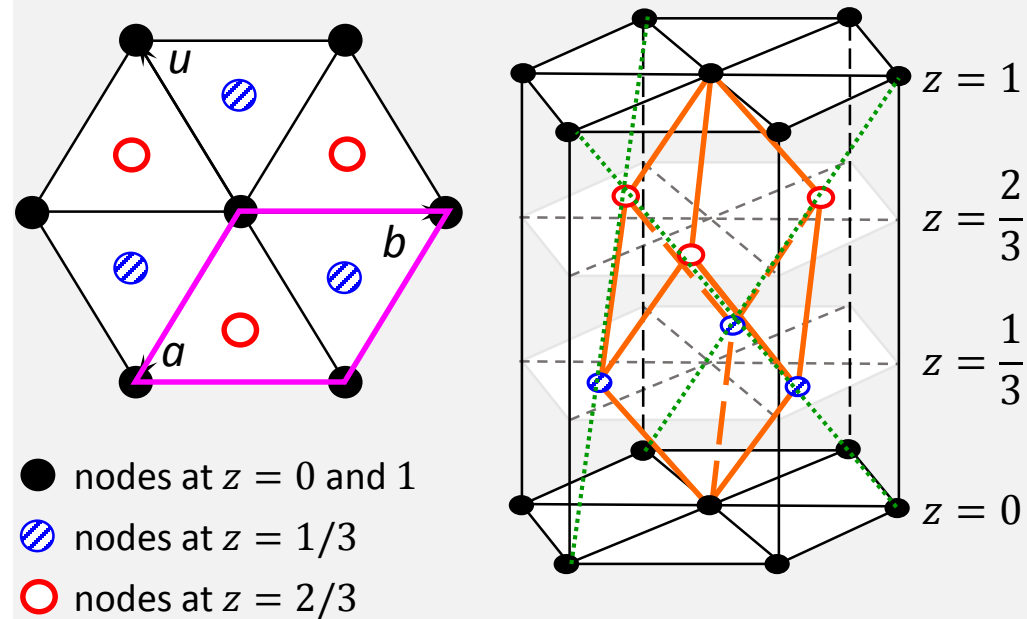
Crystal system	Point groups and Laue classes	Primary direction	Secondary direction	Tertiary direction
triclinic	1, $\bar{1}$	—	—	—
monoclinic	2, m , $2/m$	\vec{b} (ou \vec{c})	—	—
orthorhombic	222, $2mm$, mmm	\vec{a}	\vec{b}	\vec{c}
trigonal	3, $\bar{3}$ 32, $3m$, $\bar{3}m$	\vec{c}	$\vec{a}, \vec{b}, -\vec{a}-\vec{b}$	—
tetragonal or quadratic	4, $\bar{4}$, $4/m$ 422, $4mm$, $\bar{4}2m$, $4/mmm$	\vec{c}	\vec{a}, \vec{b}	$\vec{a}+\vec{b}, \vec{a}-\vec{b}$
hexagonal	6, $\bar{6}$, $6/m$ 622, $6mm$, $\bar{6}2m$, $6/mmm$	\vec{c}	$\vec{a}, \vec{b}, -\vec{a}-\vec{b}$	$2\vec{a}+\vec{b}, \dots$
cubic	23, $m\bar{3}$ 432, $\bar{4}3m$, $m\bar{3}m$	$\vec{a}, \vec{b}, \vec{c}$	$\vec{a}+\vec{b}+\vec{c}, \dots$	$\vec{a}+\vec{b}, \dots$

2. Translation Symmetry: The 14 Bravais lattices

- 6 primitive lattices (one for each of the 6 conventional cells),
- 8 non primitive ones, by adding nodes in the former cells, provided no symmetry element is lost & the centered cell is more symmetric than any primitive cell.

Symbole	Lattice mode	m
P	primitive	1
I	body centered	2
F	all face centered	4
A, B, C	A -, B -, C -face centered: $(\vec{b}, \vec{c}), (\vec{a}, \vec{c}), (\vec{a}, \vec{b})$ respectively	2
R	rhombohedrally centered: additional lattice nodes at $1/3$ and $2/3$ of the long diagonal of the h cell (\rightarrow trigonal system)	3

N.B.: the primitive cell of the hR cell is a
rhombohedral cell
($a = b = c, \alpha = \beta = \gamma \neq 90^\circ$)



2. Translation Symmetry: The 14 Bravais lattices

Conventional cell	Lattice mode				
	<i>P</i>	<i>I</i>	<i>F</i>	<i>C</i>	<i>R</i>
triclinic					
monoclinic					
orthorhombic					
tetragonal					
hexagonal					
cubic					

Reminder:

For centered cells,
 \exists additional lattice translations.

Example: *I* lattice

$$\begin{cases} \vec{T} = u\vec{a} + v\vec{b} + w\vec{c} \\ \vec{T}' = \vec{T} + \frac{1}{2}(\vec{a} + \vec{b} + \vec{c}) \end{cases}$$

with u, v, w integers

2. Translation Symmetry: Example – the diamond structure

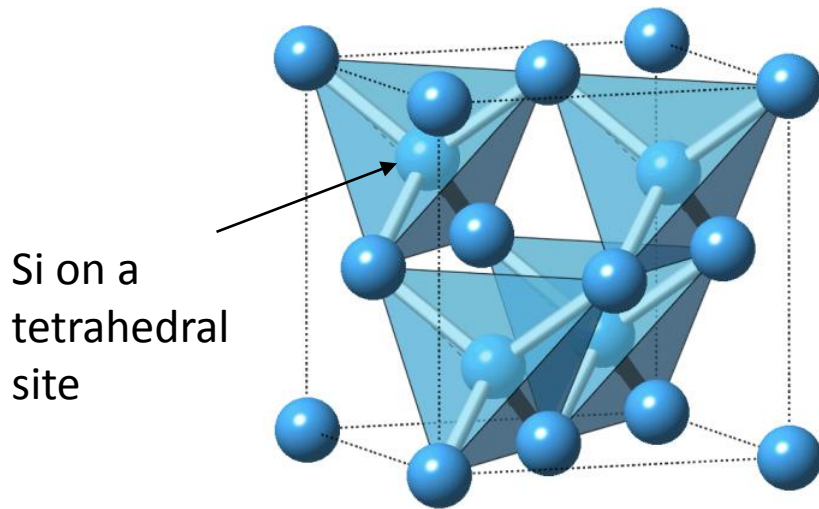
Si (diamond structure): cubic F lattice, motif = atoms at $(0,0,0)$ and $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$

F lattice ($m = 4$) \rightarrow lattice translations:

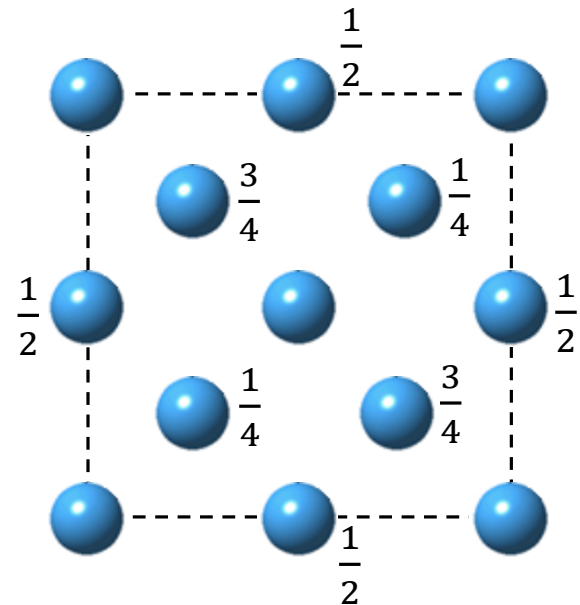
$$\vec{T}_1 = u\vec{a} + v\vec{b} + w\vec{c}, \vec{T}_2 = \vec{T}_1 + \frac{1}{2}(\vec{a} + \vec{b}), \vec{T}_3 = \vec{T}_1 + \frac{1}{2}(\vec{b} + \vec{c}), \vec{T}_4 = \vec{T}_1 + \frac{1}{2}(\vec{a} + \vec{c})$$

$\rightarrow 4 \times 2 = 8$ Si atoms per unit cell with coordinates:

$$(0,0,0), (\frac{1}{2}, \frac{1}{2}, 0), (0, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, 0, \frac{1}{2}), \text{ and } (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}), (\frac{3}{4}, \frac{3}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{3}{4}, \frac{3}{4}), (\frac{3}{4}, \frac{1}{4}, \frac{3}{4})$$



Si on a tetrahedral site



2. Translation Symmetry: Lattice directions $[uvw]$

- Family of lattices directions

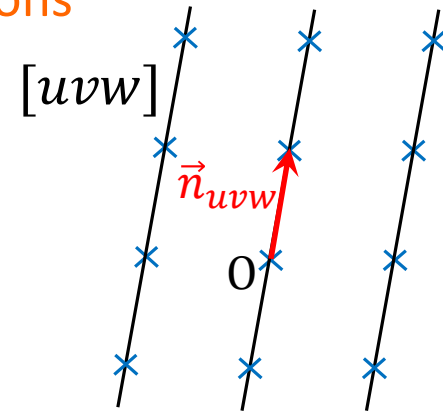
One can group all lattice nodes into parallel equidistant directions

labelled $[uvw]$ along $\vec{n}_{uvw} = u\vec{a} + v\vec{b} + w\vec{c}$

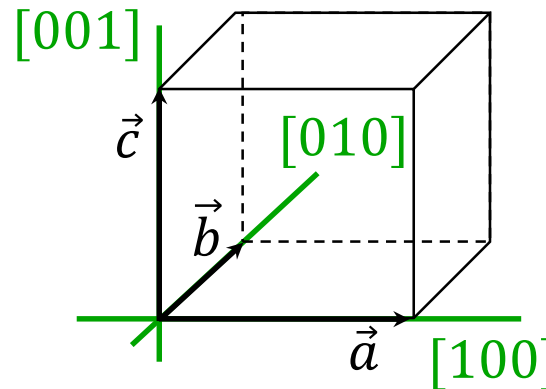
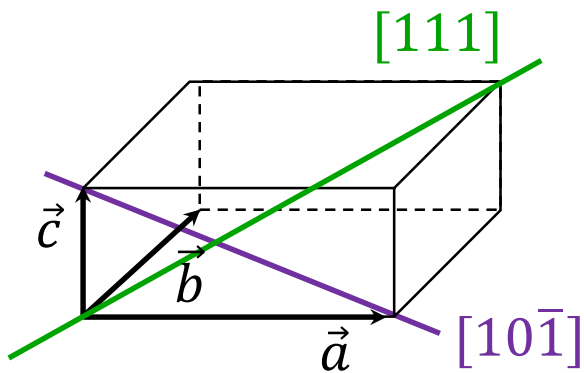
n_{uvw} (length of the direction vector): direction parameter

u, v, w (coprime integers): direction indices

A family of lattice directions contains all lattice points.



- Examples:



Cubic unit cell:
 → directions symmetrically equivalent are labeled $\langle 100 \rangle$

2. Translation Symmetry: Net planes (hkl)

- Family of net planes

One can group all lattice nodes into **parallel equidistant net planes** labelled (hkl) of equation: $hx + ky + lz = m$ with m integer (> 0 or < 0)

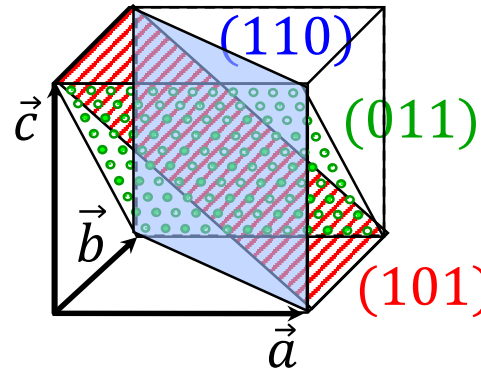
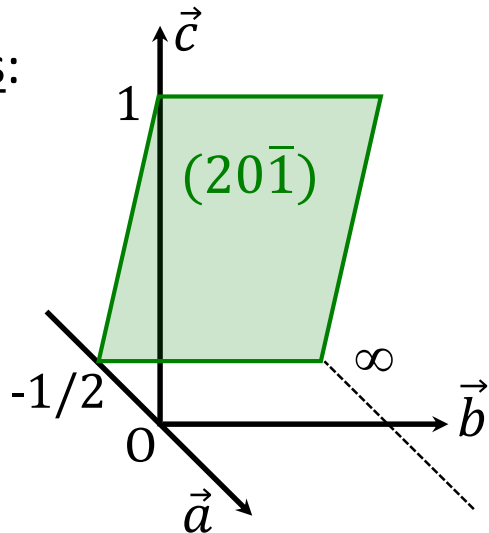
The plane the closest to the origin ($m = 1$) intercepts the \vec{a} axis at $1/h$, the \vec{b} axis at $1/k$, and the \vec{c} axis at $1/l$.

h, k, l (integers, which are coprime for a P lattice): Miller indices

d_{hkl} (distance between 2 consecutive planes): d -spacing

A family of net planes contains all lattice points.

- Examples:



Cubic unit cell:
→ planes
symmetrically
equivalent are
labeled $\{110\}$

Symmetry relations between crystals: *twinned crystals*

Twinned crystal: association of identical single-crystals with different orientations, connected through a point group symmetry: reflection, rotation, or inversion.

Formation of twinned crystals

- Growth twins: occurs during the crystal growth;
- Annealing or Transformation twins: upon cooling (phase transition)
- Deformation or gliding twins: result of stress after the crystal has formed



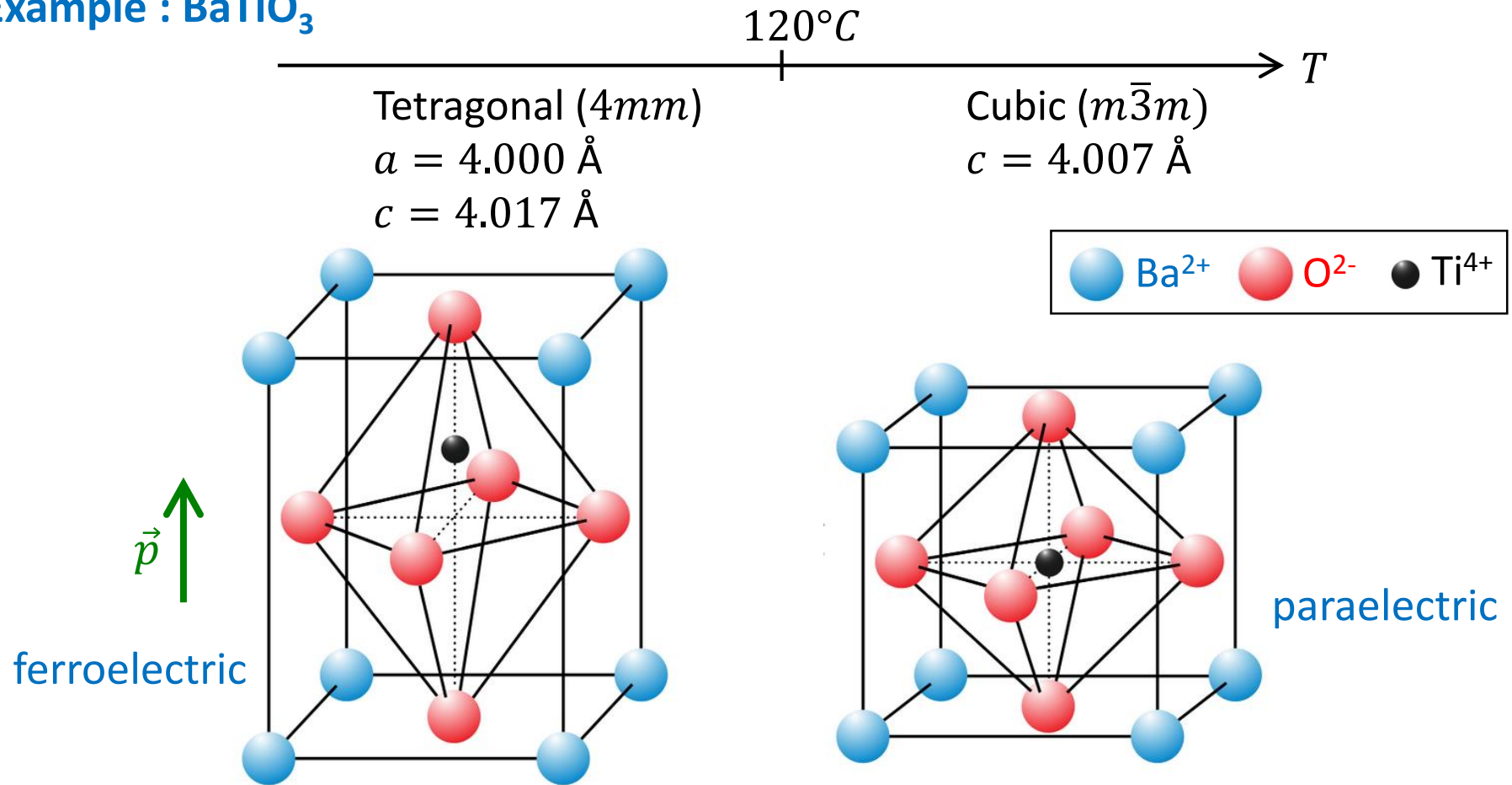
Twinned pyrite crystal



Japanese twins of quartz

Symmetry relations between crystals: *phase transitions*

Example : BaTiO₃



With no external stress (pressure, electric field, ...):

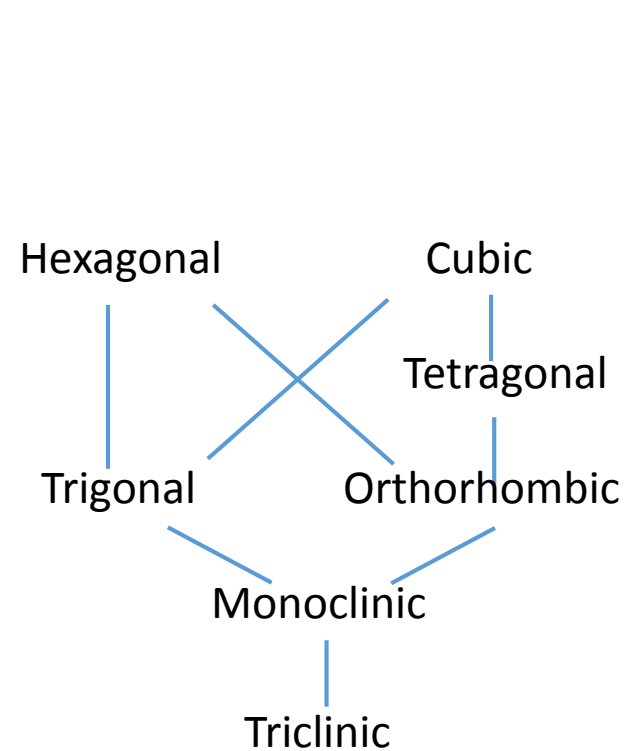
3 different twins with 2 domains at 180° each

Symmetry relations between crystals: *phase transitions*

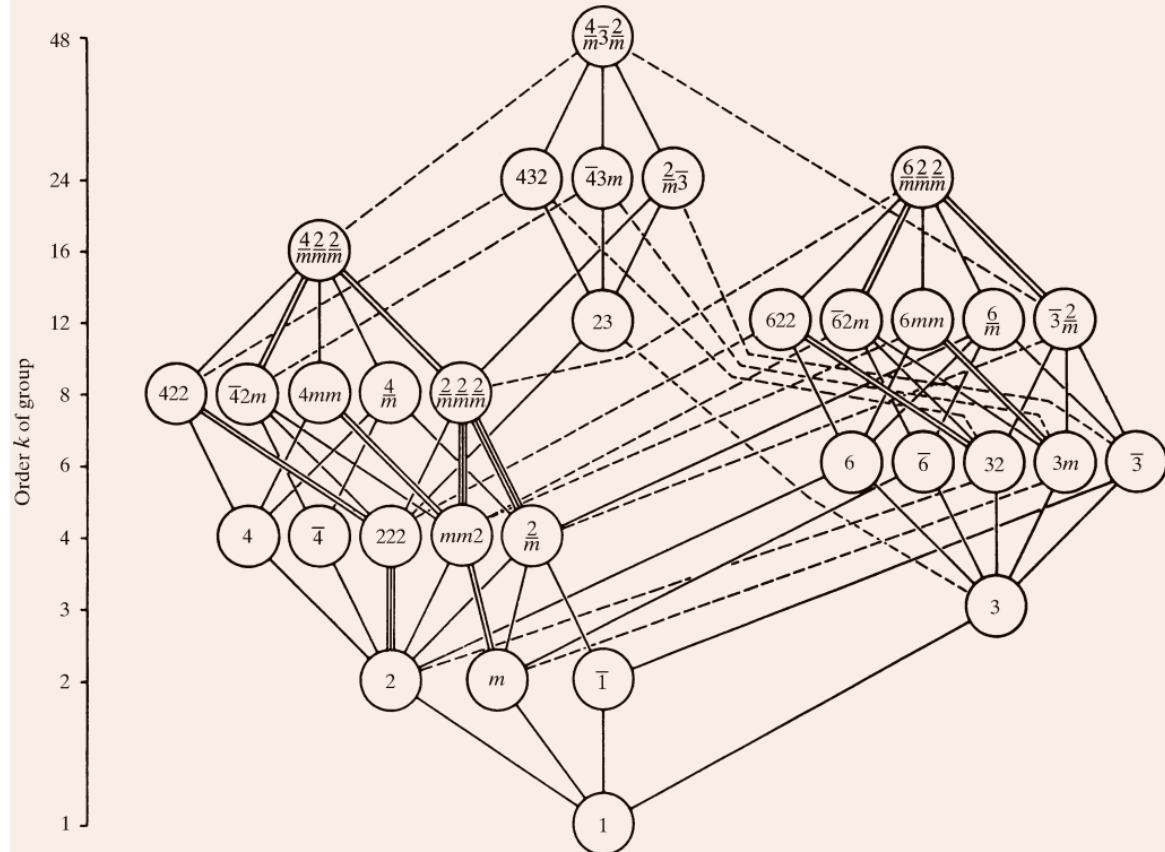
2nd order phase transition:

There exist a group / subgroup relation between the 2 phases

Example: cooling down → **symmetry lowers** (change of point group)



Relation between the 7 crystal systems



Group / subgroup relations between the 32 point groups
 Source: ITC, volume A, page 796

3. Space group symmetry

Crystal = lattice + motif

translations \vec{T}

+

∃ symmetries acting inside the motif
(symmetry planes and axes)



230 SPACE GROUPS

- Describe the symmetry of the internal structure of crystals
- Allow to classify all the crystals

14 Bravais lattices

Point symmetries
(32 point groups)
combined or not with
a fractional translation

- International Tables for Crystallography (ITC) (<https://it.iucr.org>)
- Bilbao Crystallographic Server (<http://www.cryst.ehu.es>)
- A Hypertext Book of Crystallographic Space Group Diagrams and Tables (<http://img.chem.ucl.ac.uk/sgp/mainmenu.htm>)

3. Space group symmetry



INTERNATIONAL TABLES
for CRYSTALLOGRAPHY



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International Tables for Crystallography Resources

The following resources are available as part of **International Tables Online**:

- Search for a crystallographic symmetry group

Go to No.

- Symmetry database
- Retrieve scattering factors for electron diffraction
- Plot scattering factors for electron diffraction
- Retrieve scattering lengths for neutron diffraction
- Resources for Volume D (*Tenχar* and *GI*KoBo-1*)
- Superspace Group Finder
- CIF dictionaries
 - Core CIF Dictionary
 - Electron Density CIF Dictionary
 - Image CIF Dictionary
 - Macromolecular CIF Dictionary
 - Modulated Structures CIF Dictionary
 - Powder CIF Dictionary
 - Symmetry CIF Dictionary

3. Space group symmetry

www.cryst.ehu.es

img.chem.ucl.ac.uk/sgp/mainmenu.htm

bilbao crystallographic server

Contact us

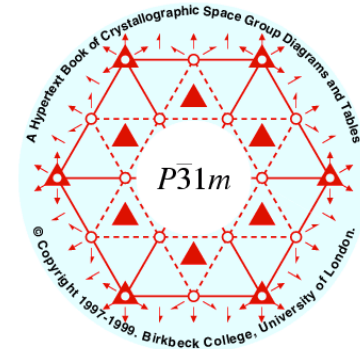
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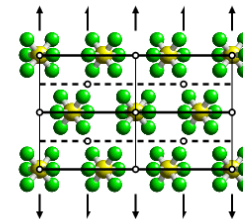
How to

Space-group symmetry

GENPOS	Generators and General Positions of Space Groups
WYCKPOS	Wyckoff Positions of Space Groups
HKLCOND	Reflection conditions of Space Groups
MAXSUB	Maximal Subgroups of Space Groups
SERIES	Series of Maximal Isomorphic Subgroups of Space Groups
WYCKSETS	Equivalent Sets of Wyckoff Positions
NORMALIZER	Normalizers of Space Groups
KVEC	The k-vector types and Brillouin zones of Space Groups
SYMMETRY OPERATIONS	Geometric interpretation of matrix column representations of symmetry operations
IDENTIFY GROUP	Identification of a Space Group from a set of generators in an arbitrary setting



A Hypertext Book of Crystallographic Space Group Diagrams and Tables



CD-ROM Cover Picture



[High-Resolution Space Group
Diagrams and Tables](#)
(1280 × 1024 pixel screens)



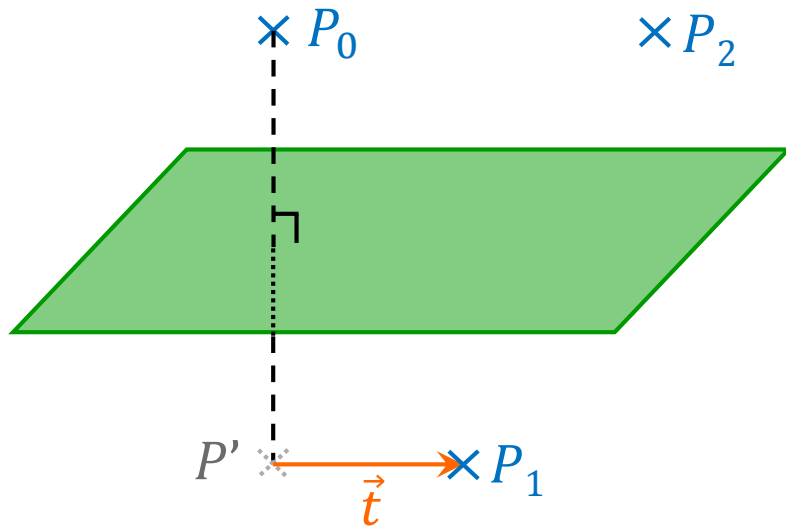
[Medium-Resolution Space Group
Diagrams and Tables](#)
(1024 × 768 pixel screens)



3. Space group symmetry: *symmetry planes*

- Glide plane

Combination of a **reflection** (through a plane) and a **fractional translation** $\vec{t} \parallel$ plane
 acting inside the unit cell



Example: glide plane $a \perp \vec{c}$ at $z = \frac{1}{4}$
 $a \times a \rightarrow$ lattice translation

$$P_0 P_2 = \vec{a} \rightarrow \boxed{\vec{t} = \frac{\vec{a}}{2}}$$

Seitz notation: $\{\alpha | \vec{t}_\alpha\} = \left\{ m_z \left| \frac{1}{2}, 0, \frac{1}{2} \right. \right\}$

4 × 4 matrix:
$$\begin{pmatrix} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1/2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

α : point symmetry

\vec{t}_α : translation embedding the glide translation + the position of α

3. Space group symmetry: symmetry planes

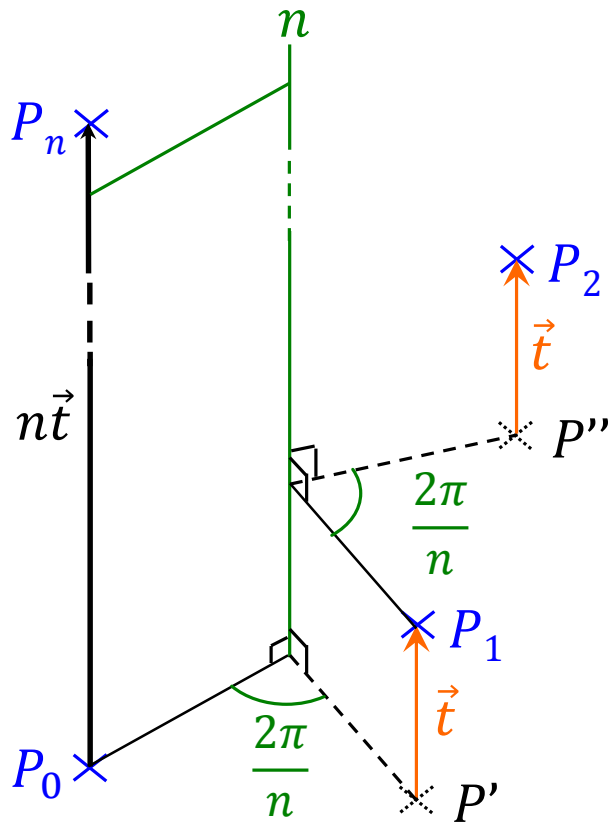
The various symmetry planes and their Hermann-Mauguin symbol

Printed symbol	Symmetry plane	Graphic symbol		Nature of the gliding (fractional translation \vec{t})
		\perp projection plane	\parallel projection plane	
m	mirror			none
a, b, c	Axial glide plane			$a/2, b/2, \text{ or } c/2$ respectively
e	Double glide plane			$a/2$ and $b/2, b/2$ and $c/2, \text{ or } a/2$ et $c/2$; OR $(a \pm b)/2$ and $c/2, \text{ etc ... for } t$ and c systems
n	Diagonal glide plane			$(a+b)/2, (b+c)/2$ or $(c+a)/2$; OR $(a+b+c)/2$ in some cases for t and c systems
d	Diamond glide plane			$(a+b)/4, (b+c)/4$ or $(c+a)/4$; OR $(a+b+c)/4$ in some cases for t and c systems

3. Space group symmetry: symmetry axes

- Screw axes

Combination of a **rotation** (around an axis n) and a **fractional translation** $\vec{t} \parallel$ **axis**



Example: screw axis $n_p \parallel \vec{c}$

$\underbrace{n_p \times \dots \times n_p}_{n \text{ times}} \rightarrow$ lattice translation

$$\overrightarrow{P_0 P_n} = n\vec{t} = p\vec{c}$$

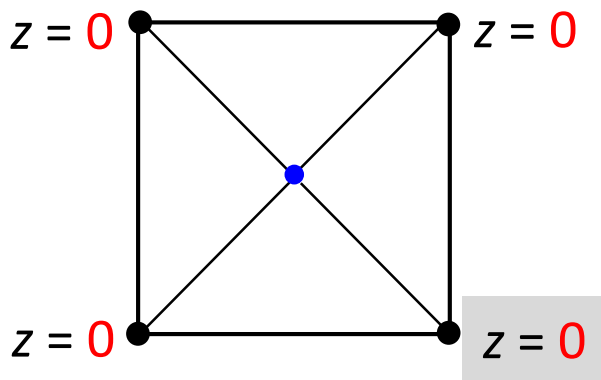
with $\begin{cases} n = 1, 2, 3, 4, \text{ or } 6 \\ p \text{ integer} < n \end{cases}$

$$\rightarrow \boxed{\vec{t} = \frac{p}{n} \vec{c}} \quad \text{with} \quad \boxed{p = 0, 1, \dots, n - 1}$$

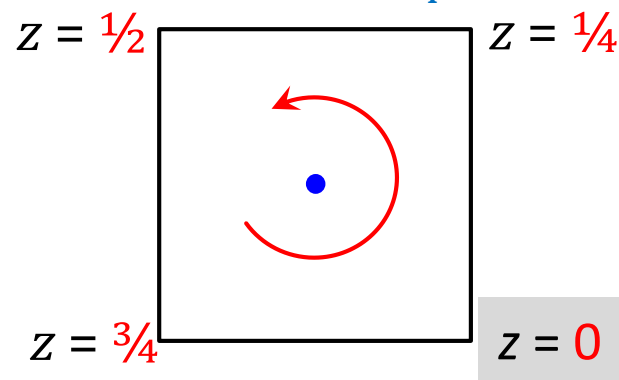
3. Space group symmetry: symmetry axes

Example: screw axes $4_p \parallel \vec{c}$

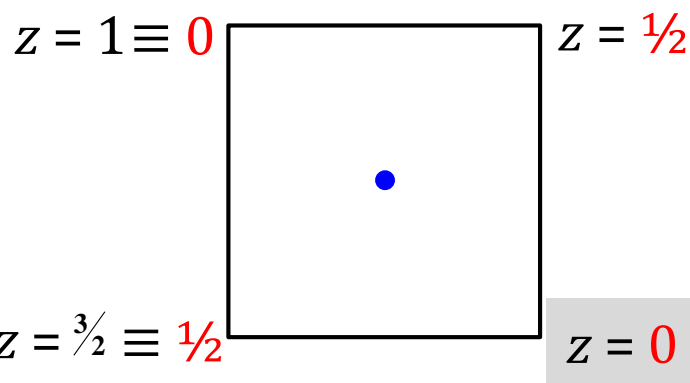
Axis 4: $\vec{t} = \vec{0}$



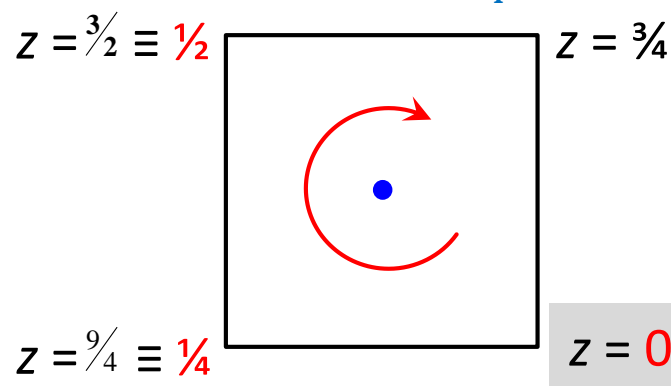
Axis 4_1 : $\vec{t} = \frac{1}{4}\vec{c}$



Axis 4_2 : $\vec{t} = \frac{1}{2}\vec{c}$



Axis 4_3 : $\vec{t} = \frac{3}{4}\vec{c}$



3. Space group symmetry: symmetry axes

The various symmetry axes and their Hermann-Mauguin symbol (*projection plane* $\perp \vec{c}$)

Printed symbol	Symmetry axis	Graphic symbol	Gliding \vec{t}	Printed symbol	Symmetry axis	Graphic symbol	Gliding \vec{t}
1	Identity	none	none	4	4-fold rotat°		none
$\bar{1}$	Inversion	o	none	4_1	4-fold screw axes		$c/4$
2	2-fold rotation axis	 (\perp plan proj.) (\parallel plan proj.)	none	4_2			$2c/4$
				4_3			$3c/4$
2_1	2-fold screw axis	 (\perp plan proj.) (\parallel plan proj.)	$c/2$ $a/2$ ou $b/2$	$\bar{4}$	4-fold rotoinversion		none
3	3-fold rotation axis	\perp plan proj. 	none	6	6-fold rotat°		none
				3_1	2-fold screw axes		$c/3$
3_2	2-fold screw axes		$2c/3$	6_1	6-fold screw axes		$c/6$
$\bar{3}$	3-fold rotoinversion		none	6_2			$2c/6$
				6_3			$3c/6$
				6_4		$4c/6$	
				6_5		$5c/6$	
				$\bar{6}$	6-fold rotoinversion		none

3. Space group symmetry: the 230 space groups

- International notation (Hermann-Mauguin symbol)

Ex. $P4_2/mmc$

1st letter : capital letter designing the **lattice mode** P, I, F, A (B or C), R
Following letters: **nature of the symmetry elements**

Symmetry axes (with n max and p min) and planes ($m > e > a > b > c > n > d$)

Along the primary, secondary, and tertiary directions: 3 non equivalent directions of symmetry (the same ones as point groups)

Conventional cell	Primary direction	Secondary direction	Tertiary direction
triclinic	A single symbol (1 or $\bar{1}$), thus no direction of symmetry		
monoclinic	A single direction of symmetry: b or c (order 2, unique axis)		
orthorhombic	a (order 2)	b (order 2)	c (order 2)
tetragonal	$[001]$ (order 4)	$\langle 100 \rangle$, i.e. a and b (order 2)	$\langle 110 \rangle$, i.e. $a \pm b$ (order 2)
hexagonal	c (order 6 or 3)	$\langle 100 \rangle$, i.e. $a, b, [1\bar{1}0]$ (order 2)	$\langle 210 \rangle$, i.e. $[210], [\bar{1}20], [1\bar{1}0]$ (order 2)
cubic	$\langle 100 \rangle$ (order 4 or 2)	$\langle 111 \rangle$ (order 3)	$\langle 110 \rangle$ (order 2)

3. Space group symmetry: the 230 space groups

cryst. point
syst. group

cryst. syst.	point group	space group No.	space group symbol		
a	1	1	$P1$		
		2	$P\bar{1}$		
	m	2	3	$P2$	
			4	$P2_1$	
			5	$C2$	
			6	Pm	
	m	2/m	7	Pc	
			8	Cm	
			9	Cc	
			10	$P2/m$	
			11	$P2_1/m$	
	o	222	12	$C2/m$	
			13	$P2/c$	
			14	$P2_1/c$	
			15	$C2/c$	
mm2			222	16	$P222$
				17	$P222_1$
				18	$P2_12_12$
				19	$P2_12_12_1$
				20	$C222_1$
				21	$C222$
				22	$F222$
				23	$I222$
				24	$I2_12_12_1$
				mm2	222
26			$Pmc2_1$		
27	$Pcc2$				
28	$Pma2$				
29	$Pca2_1$				
30	$Pnc2$				
31	$Pmn2_1$				
32	$Pba2_1$				
33	$Pna2_1$				
34	$Pnn2$				
35	$Cmm2$				
36	$Cmc2_1$				
37	$Ccc2$				
38	$Amm2$				
39	$Aem2$				
40	$Ama2$				
41	$Aea2$				
42	$Fmm2$				
43	$Fdd2$				
44	$Imm2$				
45	$Iba2$				
46	$Ima2$				
mmm	222	47	$Pnmm$		
		48	$Pnmm$		
		49	$Pccm$		
		50	$Pban$		

cryst. point
syst. group

cryst. syst.	point group	space group No.	space group symbol			
t	4	51	$Pmma$			
		52	$Pnna$			
		53	$Pmna$			
		54	$Pcca$			
		55	$Pbam$			
		56	$Pccn$			
		57	$Pbcm$			
		58	$Pnmm$			
		59	$Pmnm$			
		60	$Pbcn$			
		61	$Pbca$			
		62	$Pnna$			
		63	$Cmcm$			
		64	$Cmce$			
		65	$Cmmm$			
		66	$Cccm$			
		67	$Cmme$			
		68	$Ccce$			
		69	$Fmmm$			
		70	$Fddd$			
		71	$Immm$			
		72	$Ibam$			
		73	$Ibca$			
		74	$Imma$			
t	4	75	$P4$			
		76	$P4_1$			
		77	$P4_2$			
		78	$P4_3$			
		79	$I4$			
		80	$I4_1$			
		4	4	81	$P\bar{4}$	
				82	$I\bar{4}$	
				4/m	83	$P4/m$
					84	$P4_2/m$
		422	4	85	$P4/n$	
				86	$P4_2/n$	
87	$I4/m$					
88	$I4_1/a$					
89	$P422$					
90	$P42_12$					
91	$P4_122$					
92	$P4_12_12$					
93	$P4_222$					
94	$P4_22_12$					
95	$P4_322$					
96	$P4_32_12$					
4mm	422	97	$I422$			
		98	$I4_122$			
		99	$P4mm$			
		100	$P4bm$			

cryst. point
syst. group

cryst. syst.	point group	space group No.	space group symbol		
h	3	101	$P4_2cm$		
		102	$P4_2nm$		
		103	$P4cc$		
		104	$P4nc$		
		105	$P4_2mc$		
		106	$P4_2bc$		
		107	$I4mm$		
		108	$I4cm$		
		109	$I4_1md$		
		110	$I4_1cd$		
		42m	3	111	$P\bar{4}2m$
				112	$P\bar{4}2c$
				113	$P\bar{4}2_1m$
				114	$P\bar{4}2_1c$
				115	$P\bar{4}2m2$
				116	$P\bar{4}c2$
				117	$P\bar{4}b2$
				118	$P\bar{4}n2$
				119	$I\bar{4}m2$
				120	$I\bar{4}c2$
121	$I\bar{4}2m$				
122	$I\bar{4}2d$				
4/mmm	3	123	$P4/mmm$		
		124	$P4/mcc$		
		125	$P4/nbm$		
		126	$P4/nnc$		
		127	$P4/mbm$		
		128	$P4/mnc$		
		129	$P4/nmm$		
		130	$P4/nnc$		
		131	$P4_2/mmc$		
		132	$P4_2/mcm$		
		133	$P4_2/nbc$		
		134	$P4_2/nmm$		
135	$P4_2/mbc$				
136	$P4_2/nmm$				
137	$P4_2/nmc$				
138	$P4_2/ncm$				
139	$I4/mmm$				
140	$I4/mcm$				
141	$I4_1/amd$				
142	$I4_1/acd$				
h	3	143	$P3$		
		144	$P3_1$		
		145	$P3_2$		
		146	$R3$		
		147	$P\bar{3}$		
		148	$R\bar{3}$		
32	3	149	$P312$		
		150	$P321$		

cryst. point
syst. group

cryst. syst.	point group	space group No.	space group symbol		
h	6	151	$P3_112$		
		152	$P3_121$		
		153	$P3_212$		
		154	$P3_221$		
		155	$R32$		
		3m	6	156	$P3m1$
				157	$P31m$
				158	$P3c1$
				159	$P31c$
				160	$R3m$
				161	$R3c$
		3m	6	162	$P\bar{3}1m$
				163	$P\bar{3}1c$
				164	$P\bar{3}m1$
				165	$P\bar{3}c1$
				166	$R\bar{3}m$
				167	$R\bar{3}c$
h	6			168	$P6$
		169	$P6_1$		
		170	$P6_5$		
		171	$P6_2$		
		172	$P6_4$		
		173	$P6_3$		
		174	$P6$		
		175	$P6/m$		
		176	$P6_3/m$		
		177	$P622$		

6 conventional cells

14 Bravais lattices (translation symmetry)

32 point groups

Symmetry at the macroscopic scale

230 space groups

Symmetry at the microscopic scale

cryst. point
syst. group

cryst. syst.	point group	space group No.	space group symbol		
c	23	178	$P6_122$		
		179	$P6_522$		
		180	$P6_222$		
		181	$P6_122$		
		182	$P6_322$		
		6mm	6	183	$P6mm$
				184	$P6cc$
				185	$P6_3cm$
				186	$P6_3mc$
				187	$P\bar{6}m2$
				188	$P\bar{6}c2$
		6m2	6	189	$P\bar{6}2m$
				190	$P\bar{6}c2$
				191	$P6/mmm$
		6/mmm	6	192	$P6/mcc$
				193	$P6/mcm$
				194	$P6/mmc$
				195	$P23$
		c	23	196	$F23$
				197	$I23$
198	$P2_13$				
199	$I2_13$				
200	$Pm\bar{3}$				
201	$Pn\bar{3}$				
202	$Fm\bar{3}$				
203	$Fd\bar{3}$				
204	$Im\bar{3}$				

cryst. point
syst. group

cryst. syst.	point group	space group No.	space group symbol		
432	23	205	$Pa\bar{3}$		
		206	$Ia\bar{3}$		
		207	$P432$		
		208	$P4_332$		
		209	$F432$		
		210	$F4_132$		
		211	$I432$		
		212	$P4_332$		
		213	$P4_132$		
		214	$I4_132$		
		43m	23	215	$P4_3m$
				216	$F4_3m$
				217	$I4_3m$
				218	$P4_3n$
				219	$F4_3c$
220	$I4_3d$				
m3m	23	221	$Pm\bar{3}m$		
		222	$Pn\bar{3}n$		
		223	$Pm\bar{3}n$		
		224	$Pn\bar{3}m$		
		225	$Fm\bar{3}m$		
		226	$Fm\bar{3}c$		
		227	$Fd\bar{3}m$		
		228	$Fd\bar{3}c$		
		229	$Im\bar{3}m$		
		230	$Ia\bar{3}d$		

3. Space group symmetry: space group $Pnma$ – ITC

Non symmorphic SG
(\exists glide translations)

Diagrams of symmetry operations

$\triangle!$ \exists different settings
(permutations of a, b, c)

Diagrams of equivalent positions

Location of the origin

Symmetry operations:
(number), nature, location

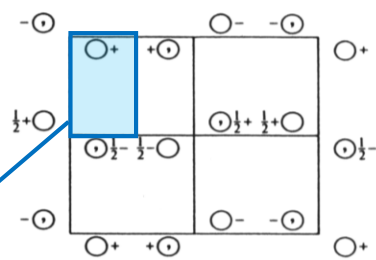
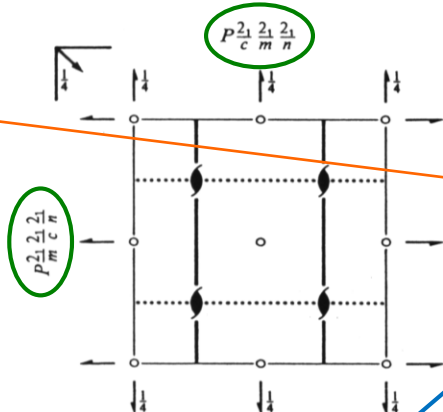
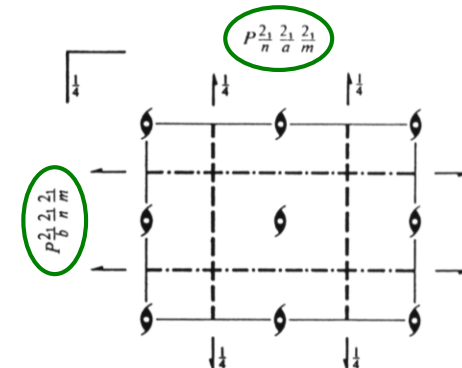
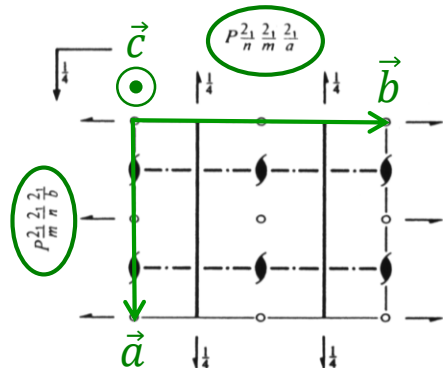
page 1/2 of $Pnma$
taken from the ITC,
volume A



$Pnma$
No. 62

D_{2h}^{16}
 $P 2_1/n 2_1/m 2_1/a$

mmm Orthorhombic
Patterson symmetry $Pmmm$



Origin at $\bar{1}$ on $12_1 1$

Asymmetric unit $0 \leq x \leq \frac{1}{2}; 0 \leq y \leq \frac{1}{2}; 0 \leq z \leq 1$

Symmetry operations

- | | | | |
|-----------------------|--|----------------------------------|--|
| (1) 1 | (2) $2(0,0,\frac{1}{2})$ $\frac{1}{2},0,z$ | (3) $2(0,\frac{1}{2},0)$ $0,y,0$ | (4) $2(\frac{1}{2},0,0)$ $x,\frac{1}{2},z$ |
| (5) $\bar{1}$ $0,0,0$ | (6) a $x,y,\frac{1}{2}$ | (7) m $x,\frac{1}{2},z$ | (8) $n(0,\frac{1}{2},\frac{1}{2})$ $\frac{1}{2},y,z$ |

Smallest volume of the unit cell containing all structural information

3. Space group symmetry: space group $Pnma - ITC$

Orthorhombic

$P2_1/n 2_1/m 2_1/a$

Bravais lattice

Screw axis $2_1 \parallel \vec{a}$
Glide plane $n \perp \vec{a}$

Screw axis $2_1 \parallel \vec{c}$
Glide plane $a \perp \vec{c}$

Screw axis $2_1 \parallel \vec{b}$
Mirror plane $m \perp \vec{b}$

Point group: $\frac{2}{m} \frac{2}{m} \frac{2}{m}$

($n, m, a \rightarrow m$ and $2_1 \rightarrow 2$)

"Representation" of the symmetry operations

Symmetry operations

(1) 1
(5) $\bar{1}$ $0,0,0$

(2) $2(0,0,\frac{1}{2})$ $\frac{1}{2},0,z$
(6) a $x,y,\frac{1}{2}$

(3) $2(0,\frac{1}{2},0)$ $0,y,0$
(7) m $x,\frac{1}{2},z$

(4) $2(\frac{1}{2},0,0)$ $x,\frac{1}{2},\frac{1}{2}$
(8) $n(0,\frac{1}{2},\frac{1}{2})$ $\frac{1}{2},y,z$

2-fold rotation
followed by $\vec{t} = \frac{1}{2}\vec{c}$
i.e. axis $2_1 \parallel \vec{c}$

axis $\parallel \vec{c}$ at
 $x = \frac{1}{4}$ et $y = 0$

Glide plane n
with $\vec{t} = \frac{1}{2}(\vec{b} + \vec{c})$

plane (x, y) , i.e. $\perp \vec{a}$
with $x = \frac{1}{4}$

3. Space group symmetry: space group $Pnma - ITC$

CONTINUED

No. 62

$Pnma$

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (3); (5)

Arbitrary choice of generators for the SG

Identity
Elementary translations
+ some of the symmetry axes and planes

Positions		Coordinates							
Multiplicity, Wyckoff letter, Site symmetry		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
8	d 1	x, y, z	$\bar{x} + \frac{1}{2}, \bar{y}, z + \frac{1}{2}$	$\bar{x}, y + \frac{1}{2}, \bar{z}$	$x + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{z} + \frac{1}{2}$	$\bar{x}, \bar{y}, \bar{z}$	$x + \frac{1}{2}, y, \bar{z} + \frac{1}{2}$	$x, \bar{y} + \frac{1}{2}, z$	$\bar{x} + \frac{1}{2}, y + \frac{1}{2}, z + \frac{1}{2}$
4	c .m.	$x, \frac{1}{2}, z$	$\bar{x} + \frac{1}{2}, \frac{1}{2}, z + \frac{1}{2}$	$\bar{x}, \frac{1}{2}, \bar{z}$	$x + \frac{1}{2}, \frac{1}{2}, \bar{z} + \frac{1}{2}$				
4	b $\bar{1}$	$0, 0, \frac{1}{2}$	$\frac{1}{2}, 0, 0$	$0, \frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}, 0$				
4	a $\bar{1}$	$0, 0, 0$	$\frac{1}{2}, 0, \frac{1}{2}$	$0, \frac{1}{2}, 0$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$				

Reflection conditions

General:

- $0kl : k+l = 2n$
- $hk0 : h = 2n$
- $h00 : h = 2n$
- $0k0 : k = 2n$
- $00l : l = 2n$

Special: as above, plus no extra conditions

- $hkl : h+l, k = 2n$
- $hkl : h+l, k = 2n$

Symmetry of special projections

Along [001] $p2gm$
 $a' = \frac{1}{2}a$ $b' = b$
Origin at $0, 0, z$

Along [100] $c2mm$
 $a' = b$ $b' = c$
Origin at $x, \frac{1}{2}, \frac{1}{2}$

Along [010] $p2gg$
 $a' = c$ $b' = a$
Origin at $0, y, 0$

Maximal non-isomorphic subgroups

- I** [2] $P2_12_12_1$ 1; 2; 3; 4
- [2] $P112_1/a(P2_1/c)$ 1; 2; 5; 6
- [2] $P12_1/m1(P2_1/m)$ 1; 3; 5; 7
- [2] $P2_1/n11(P2_1/c)$ 1; 4; 5; 8
- [2] $Pnm2_1(Pmn2_1)$ 1; 2; 7; 8
- [2] $Pn2_1a(Pna2_1)$ 1; 3; 6; 8
- [2] $P2_1ma(Pmc2_1)$ 1; 4; 6; 7

IIa none

IIb none

Maximal isomorphic subgroups of lowest index

- IIc** [3] $Pnma(a' = 3a)$; [3] $Pnma(b' = 3b)$; [3] $Pnma(c' = 3c)$

Minimal non-isomorphic supergroups

I none

- II** [2] $Amma(Cmcm)$; [2] $Bbmm(Cmcm)$; [2] $Ccmb(Cmca)$; [2] $Imma$; [2] $Pnmm(2a' = a)(Pmnn)$; [2] $Pcma(2b' = b)(Pbam)$; [2] $Pbma(2c' = c)(Pbcm)$

See Lecture III

subgroups / supergroups,
for crystallogr. transitions to a
lower / higher symmetry phase
See Lecture II

See also www.cryst.ehu.es

page 2/2 of $Pnma$
taken from the ITC,
volume A



3. Space group symmetry: *space group Pnma – ITC*

Wyckoff sites: List of the different sites from the most general (*i.e.* less symmetrical) to the less general position (*i.e.* most symmetrical: special position)

Positions			Coordinates			
Multiplicity, Wyckoff letter, Site symmetry						
8	<i>d</i>	1	(1) x, y, z (5) $\bar{x}, \bar{y}, \bar{z}$	(2) $\bar{x} + \frac{1}{2}, \bar{y}, z + \frac{1}{2}$ (6) $x + \frac{1}{2}, y, \bar{z} + \frac{1}{2}$	(3) $\bar{x}, y + \frac{1}{2}, \bar{z}$ (7) $x, \bar{y} + \frac{1}{2}, z$	(4) $x + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{z} + \frac{1}{2}$ (8) $\bar{x} + \frac{1}{2}, y + \frac{1}{2}, z + \frac{1}{2}$
4	<i>c</i>	$.m.$	$x, \frac{1}{2}, z$	$\bar{x} + \frac{1}{2}, \frac{1}{2}, z + \frac{1}{2}$	$\bar{x}, \frac{1}{2}, \bar{z}$	$x + \frac{1}{2}, \frac{1}{2}, \bar{z} + \frac{1}{2}$
4	<i>b</i>	$\bar{1}$	$0, 0, \frac{1}{2}$	$\frac{1}{2}, 0, 0$	$0, \frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}, 0$
4	<i>a</i>	$\bar{1}$	$0, 0, 0$	$\frac{1}{2}, 0, \frac{1}{2}$	$0, \frac{1}{2}, 0$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

Site name

Multiplicity of the site

Wyckoff letter

Symmetry of the site

Coordinates of all equivalent positions

3. Space group symmetry: *space group Pnma – ITC*

Wyckoff sites: List of the different sites from the most general (*i.e.* less symmetrical) to the less general position (*i.e.* most symmetrical: special position)

Positions		Coordinates				
Multiplicity, Wyckoff letter, Site symmetry						
8	<i>d</i>	1	(1) x, y, z	(2) $\bar{x} + \frac{1}{2}, \bar{y}, z + \frac{1}{2}$	(3) $\bar{x}, y + \frac{1}{2}, \bar{z}$	(4) $x + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{z} + \frac{1}{2}$
			(5) $\bar{x}, \bar{y}, \bar{z}$	(6) $x + \frac{1}{2}, y, \bar{z} + \frac{1}{2}$	(7) $x, \bar{y} + \frac{1}{2}, z$	(8) $\bar{x} + \frac{1}{2}, y + \frac{1}{2}, z + \frac{1}{2}$

Symmetry operations

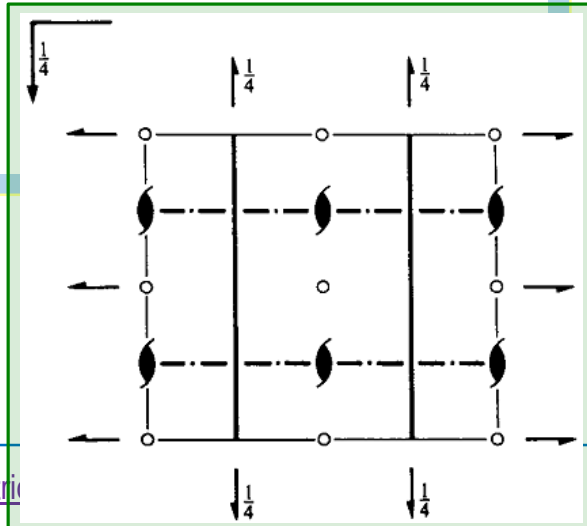
(1) 1	(2) 2(0,0, $\frac{1}{2}$) $\frac{1}{2}, 0, z$	(3) 2(0, $\frac{1}{2}$,0) $0, y, 0$	(4) 2($\frac{1}{2}$,0,0) $x, \frac{1}{2}, \frac{1}{2}$
(5) $\bar{1}$ 0,0,0	(6) <i>a</i> $x, y, \frac{1}{2}$	(7) <i>m</i> $x, \frac{1}{2}, z$	(8) <i>n</i> (0, $\frac{1}{2}$, $\frac{1}{2}$) $\frac{1}{2}, y, z$

3. Space group symmetry: *space group Pnma* – ITC

Wyckoff sites: List of the different sites from the most general (*i.e.* less symmetrical) to the less general position (*i.e.* most symmetrical: special position)

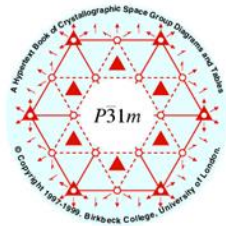
Positions		Coordinates				
Multiplicity	Wyckoff letter	Site symmetry				
8	<i>d</i>	1	(1) x, y, z	(2) $\bar{x} + \frac{1}{2}, \bar{y}, z + \frac{1}{2}$	(3) $\bar{x}, y + \frac{1}{2}, \bar{z}$	(4) $x + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{z} + \frac{1}{2}$
			(5) $\bar{x}, \bar{y}, \bar{z}$	(6) $x + \frac{1}{2}, y, \bar{z} + \frac{1}{2}$	(7) $x, \bar{y} + \frac{1}{2}, z$	(8) $\bar{x} + \frac{1}{2}, y + \frac{1}{2}, z + \frac{1}{2}$
4	<i>c</i>	\bar{m}	$x, \frac{1}{2}, z$	$\bar{x} + \frac{1}{2}, \frac{1}{2}, z + \frac{1}{2}$	$\bar{x}, \frac{1}{2}, \bar{z}$	$x + \frac{1}{2}, \frac{1}{2}, \bar{z} + \frac{1}{2}$
4	<i>b</i>	$\bar{1}$	$0, 0, \frac{1}{2}$	$\frac{1}{2}, 0, 0$	$0, \frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}, 0$
4	<i>a</i>	$\bar{1}$	$0, 0, 0$	$\frac{1}{2}, 0, \frac{1}{2}$	$0, \frac{1}{2}, 0$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

Origin at $\bar{1}$ on $12_1 1$



3. Space group symmetry: *Pnma*

<http://img.chem.ucl.ac.uk/sgp/>



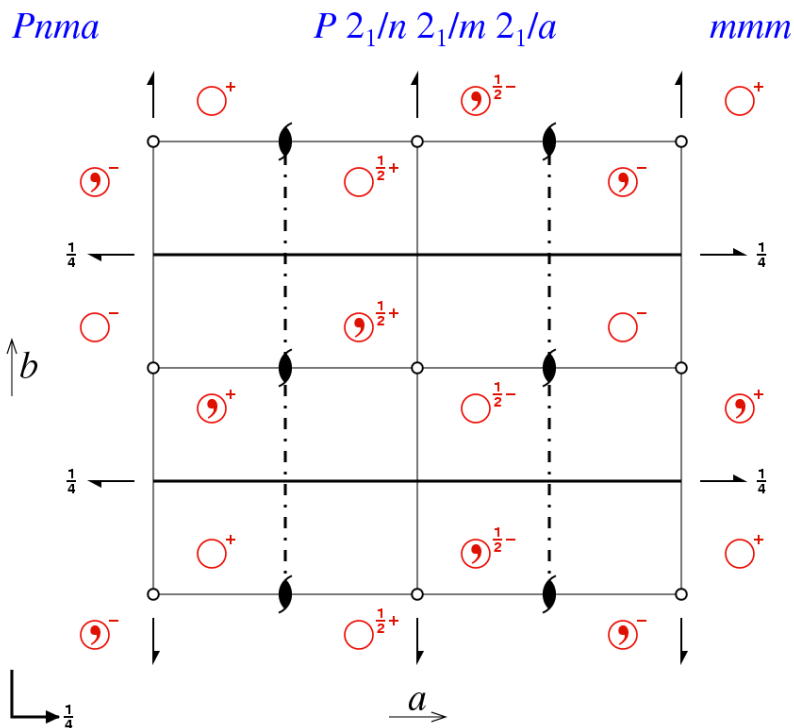
High-Resolution Space Group Diagrams and Tables

[Return](#) link to the main menu

Orthorhombic

(For a fuller list with alternative axes and origins click [here](#))

- | | | | | |
|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 61. Pbca | 62. Pnma | 63. Cmcm | 64. Cmca | 65. Cmmm |
| 66. Cccm | 67. Cmma | 68. Ccca | 69. Fmmm | 70. Fddd |
| 71. Immm | 72. Ibam | 73. Ibca | 74. Imma | |



No. 62



Symmetry Operators

- | | | | |
|---|---|-----------|---|
| 1 | x, y, z | 1 | |
| 2 | $\frac{1}{2} - x, \frac{1}{2} + y, \frac{1}{2} + z$ | <i>n</i> | $(\frac{1}{4}, y, z) [0, \frac{1}{2}, \frac{1}{2}]$ |
| 3 | $x, \frac{1}{2} - y, z$ | <i>m</i> | $(x, \frac{1}{4}, z)$ |
| 4 | $\frac{1}{2} + x, y, \frac{1}{2} - z$ | <i>a</i> | $(x, y, \frac{1}{4}) [\frac{1}{2}, 0, 0]$ |
| 5 | $\bar{x}, \bar{y}, \bar{z}$ | $\bar{1}$ | $(0, 0, 0)$ |
| 6 | $\frac{1}{2} + x, \frac{1}{2} - y, \frac{1}{2} - z$ | 2_1 | $(x, \frac{1}{4}, \frac{1}{4}) [\frac{1}{2}, 0, 0]$ |
| 7 | $\bar{x}, \frac{1}{2} + y, \bar{z}$ | 2_1 | $(0, y, 0) [0, \frac{1}{2}, 0]$ |
| 8 | $\frac{1}{2} - x, \bar{y}, \frac{1}{2} + z$ | 2_1 | $(\frac{1}{4}, 0, z) [0, 0, \frac{1}{2}]$ |

Careful: different order as compared to the ITC!

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3. Space group symmetry: *space group Pnma*

Space-group symmetry

WYCKPOS

Wyckoff Positions of Group 62 (*Pnma*)

Multiplicity	Wyckoff letter	Site symmetry	Coordinates
8	d	1	(x,y,z) (-x+1/2,-y,z+1/2) (-x,y+1/2,-z) (x+1/2,-y+1/2,-z+1/2) (-x,-y,-z) (x+1/2,y,-z+1/2) (x,-y+1/2,z) (-x+1/2,y+1/2,z+1/2)
4	c	.m.	(x,1/4,z) (-x+1/2,3/4,z+1/2) (-x,3/4,-z) (x+1/2,1/4,-z+1/2)
4	b	-1	(0,0,1/2) (1/2,0,0) (0,1/2,1/2) (1/2,1/2,0)
4	a	-1	(0,0,0) (1/2,0,1/2) (0,1/2,0) (1/2,1/2,1/2)

Wyckoff position and site symmetry group of a specific point

Specify the point by its relative coordinates (in fractions or decimals)
Variable parameters (x,y,z) are also accepted

x = y = z =

<http://www.cryst.ehu.es/>

If you want to see the Wyckoff position in other setting, [click here](#)

3. Space group symmetry: *space group Pnma*



- 8 d 1 (1) x, y, z (2) $\bar{x} + \frac{1}{2}, \bar{y}, z + \frac{1}{2}$ (3) $\bar{x}, y + \frac{1}{2}, \bar{z}$ (4) $x + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{z} + \frac{1}{2}$
 (5) $\bar{x}, \bar{y}, \bar{z}$ (6) $x + \frac{1}{2}, y, \bar{z} + \frac{1}{2}$ (7) $x, \bar{y} + \frac{1}{2}, z$ (8) $\bar{x} + \frac{1}{2}, y + \frac{1}{2}, z + \frac{1}{2}$

Symmetry operations

- (1) 1 (2) $2(0, 0, \frac{1}{2}) \frac{1}{2}, 0, z$ (3) $2(0, \frac{1}{2}, 0) 0, y, 0$ (4) $2(\frac{1}{2}, 0, 0) x, \frac{1}{2}, \frac{1}{2}$
 (5) $\bar{1} 0, 0, 0$ (6) $a x, y, \frac{1}{2}$ (7) $m x, \frac{1}{2}, z$ (8) $n(0, \frac{1}{2}, \frac{1}{2}) \frac{1}{2}, y, z$

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Space-group symmetry

General Positions of the Group 62 (*Pnma*)

GENPOS

[Click here to get the general positions in text format](#)

No.	(x,y,z) form	Matrix form	Symmetry operation	
			ITA	Seitz ?
1	x, y, z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	1	{ 1 0 }
2	$-x + 1/2, -y, z + 1/2$	$\begin{pmatrix} -1 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$	$2(0, 0, 1/2) 1/4, 0, z$	{ $2_{001} 1/2 0 1/2$ }

Same order

3. Space group symmetry: space group $Pnma - LaMnO_3$

Example: $LaMnO_3$ (space group $Pnma$)

($\equiv Pbnm$ if $\vec{a} \rightarrow \vec{b} \rightarrow \vec{c} \rightarrow \vec{a}$)

	x	y	z
La	0.518	0.25	0.007
Mn	0	0	0
O_1	-0.005	0.25	0.075
O_2	0.288	0.096	0.226

$\rightarrow 4c$

\rightarrow Motif = $La_4Mn_4O_{12}$

$\rightarrow 4a$

$\rightarrow 4c$

$\rightarrow 8d$

7 coordinates to determine out of $(4+4+12) \times 3 = 60$!!!

Positions

Multiplicity,
Wyckoff letter,
Site symmetry

Coordinates

O_2	8	d	1	(1) x, y, z	(2) $\bar{x} + \frac{1}{2}, \bar{y}, z + \frac{1}{2}$	(3) $\bar{x}, y + \frac{1}{2}, \bar{z}$	(4) $x + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{z} + \frac{1}{2}$
				(5) $\bar{x}, \bar{y}, \bar{z}$	(6) $x + \frac{1}{2}, y, \bar{z} + \frac{1}{2}$	(7) $x, \bar{y} + \frac{1}{2}, z$	(8) $\bar{x} + \frac{1}{2}, y + \frac{1}{2}, z + \frac{1}{2}$

La, O_1	4	c	$.m$	$x, \frac{1}{2}, z$	$\bar{x} + \frac{1}{2}, \frac{1}{2}, z + \frac{1}{2}$	$\bar{x}, \frac{1}{2}, \bar{z}$	$x + \frac{1}{2}, \frac{1}{2}, \bar{z} + \frac{1}{2}$
	4	b	$\bar{1}$	$0, 0, \frac{1}{2}$	$\frac{1}{2}, 0, 0$	$0, \frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}, 0$
Mn	4	a	$\bar{1}$	$0, 0, 0$	$\frac{1}{2}, 0, \frac{1}{2}$	$0, \frac{1}{2}, 0$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

3. Space group symmetry: *space group Pnma* – *LaMnO₃*



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<http://www.crystallography.net/cod/search.html>

3. Space group symmetry: *space group Pnma* – LaMnO_3

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# http://www.crystallography.net/  
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O2 O-2 0.194 0.537 0.223 1 0.0  
La1 La+3 0.0495 0.25 -0.083 1 0.0  
O1 O-2 0.474 0.25 0.07 1 0.0
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3. Space group symmetry: space group $Pnma$ – $LaMnO_3$

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ICSD for WWW

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 Elements:
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 Remarks:

Years:
 Element Count:
 Laue Class:
 Min. Distance:

Journal:
 Chem/Mineral Name:
 Centering:
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Title/Comment:
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 Distance Range:

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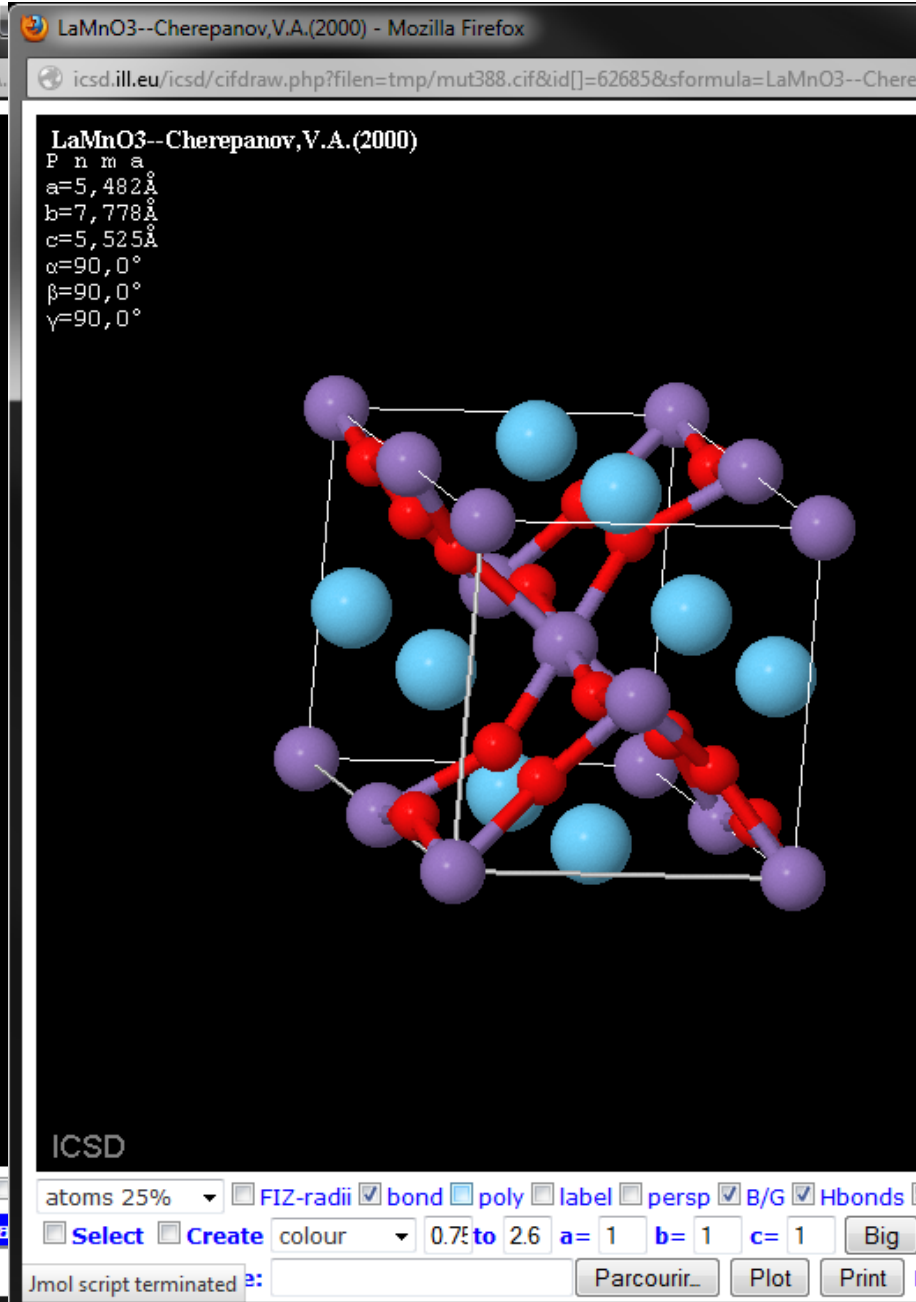
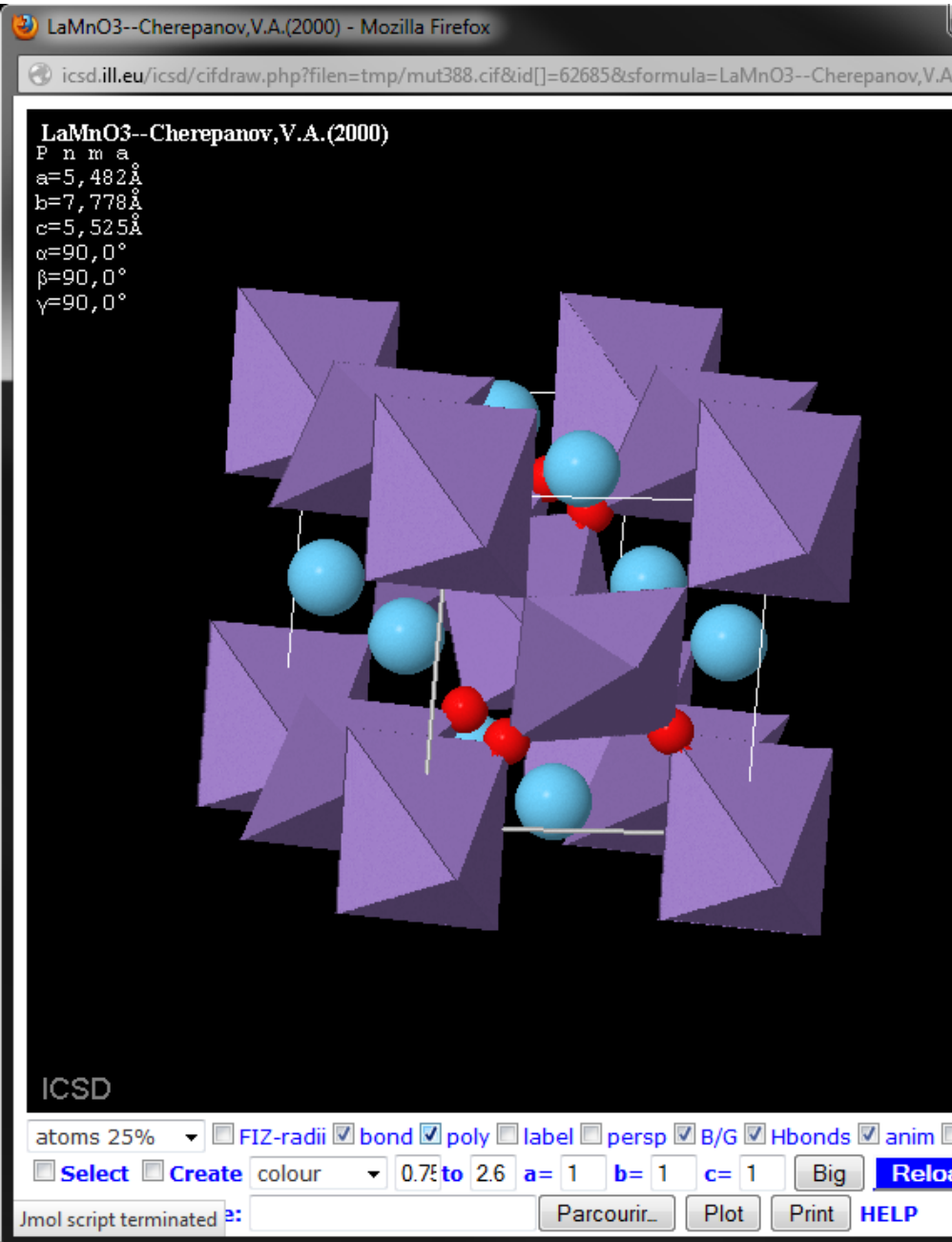
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Year	Authors	Title	Struct. Formula	sgr	Mineral
<input type="checkbox"/> 2004	Hansteen, O.H.; Breard, Y.; Fjellvag, H.; Hauback, B.C.;	Divalent manganese in reduced La Mn O3-d - effect of oxygen nonstoichiometry on structural and magnetic properties	La (Mn O2.78)	PNMA	
<input type="checkbox"/> 2000	Cherepanov, V.A.; Filonova, E.A.; Voronin, V.I.; Berger, I.F.;	Phase equilibria in the (La Co O3) - (La Mn O3) -(Ba Co Oz) (Ba Mn O3) system	La Mn O3	PNMA	
<input type="checkbox"/> 1999	Taguchi, H.; Matsu-ura, S.-I.; Nagao, M.; Kido, H.;	Electrical properties of perovskite-type La (Cr1-x Mnx) O3+d	La0.951 Mn0.951 O3	R3-CR	
<input type="checkbox"/> 1997	Ferris, V.; Goglio, G.; Brohan, L.; Joubert, O.; Molinie, P.; Ganne, M.; Dordor, P.;	Transport properties and magnetic behavior in the polycrystalline lanthanum-deficient manganate perovskite (La(1-x) Mn O3)	La0.91 (Mn0.99 O3)	R3CR	
<input type="checkbox"/> 1997	Alonso, J.A.; Martinez-Lopez, M.J.; Casais, M.T.; MacManus-Driscoll, J.L.; de Silva, P.S.I.P.N.; Cohen, L.F.; Fernandez-Diaz, M.T.;	Non-stoichiometry, structural defects and properties of La Mn O3+d with high d values (0.11)	La0.969 Mn0.93 O3	R3-CH	
<input type="checkbox"/> 1996	Shimura, T.; Hayashi, T.; Inaguma, Y.; Itoh, M.;	Magnetic and electrical properties of Lay Ax Mnx O3 (A = Na, K, Rb and Sr) with perovskite-type structure	La0.953 Mn0.935 O3	R3-CH	
<input type="checkbox"/> 1996	Hauback, B.C.; Fjellvag, H.; Sakai, N.;	Effect of nonstoichiometry on properties of La1-t Mn O3+d. III. Magnetic order studied by neutron powder diffraction	La0.92 Mn O2.88	PNMA	
<input type="checkbox"/> 1996	Hauback, B.C.; Fjellvag, H.; Sakai, N.;	Effect of nonstoichiometry on properties of La1-x Mn O3+delta III. Magnetic order studied by powder neutron diffraction	La0.88 Mn O2.82	PNMA	

Page : [1](8 results) 10 results per page.

3. Space group symmetry: *space group Pnma* – LaMnO_3



3. Space group symmetry: *space group Pnma* – LaMnO_3

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Year	Authors	Title	Struct. Formula	sgr	Mineral
2004	Hansteen, O.H.; Breard, Y.; Fjellvag, H.; Hauback, B.C.;	Divalent manganese in reduced La Mn O _{3-d} - effect of oxygen nonstoichiometry on structural and magnetic properties	La (Mn O _{2.78})	PNMA	
2000	Cherepanov, V.A.; Filonova, E.A.; Voronin, V.I.; Berger, I.F.;	Phase equilibria in the (La Co O ₃) - (La Mn O ₃) - (Ba Co O ₂) (Ba Mn O ₃) system	La Mn O ₃	PNMA	
1999	Taguchi, H.; Matsu-ura, S.-I.; Nagao, M.; Kido, H.;	Electrical properties of perovskite-type La (Cr _{1-x} Mn _x) O _{3+d}	La _{0.951} Mn _{0.951} O ₃	R3-CR	
1997	Ferris, V.; Goglio, G.; Brohan, L.; Joubert, O.; Molinie, P.; Ganne, M.; Dordor, P.;	Transport properties and magnetic behavior in the polycrystalline lanthanum-deficient manganate perovskite (La(1-x) Mn O ₃)	La _{0.91} (Mn _{0.99} O ₃)	R3CR	
1997	Alonso, J.A.; Martinez-Lopez, M.J.; Casais, M.T.; MacManus-Driscoll, J.L.; de Silva, P.S.I.P.N.; Cohen, L.F.; Fernandez-Diaz, M.T.;	Non-stoichiometry, structural defects and properties of La Mn O _{3+d} with high d values (0.11)	La _{0.969} Mn _{0.93} O ₃	R3-CH	
1996	Shimura, T.; Hayashi, T.; Inaguma, Y.; Itoh, M.;	Magnetic and electrical properties of La _x Mn _w O ₃ (A = Na, K, Rb and Sr) with perovskite-type structure	La _{0.953} Mn _{0.935} O ₃	R3-CH	
1996	Hauback, B.C.; Fjellvag, H.; Sakai, N.;	Effect of nonstoichiometry on properties of La _{1-t} Mn O _{3+d} . III. Magnetic order studied by neutron powder diffraction	La _{0.92} Mn O _{2.88}	PNMA	
1996	Hauback, B.C.; Fjellvag, H.; Sakai, N.;	Effect of nonstoichiometry on properties of La _{1-x} Mn O _{3+delta} III. Magnetic order studied by powder neutron diffraction	La _{0.88} Mn O _{2.82}	PNMA	

Page : [1](8 results) 10 results per page.

3. Space group symmetry: space group $Pnma$ – $LaMnO_3$

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Title	Phase equilibria in the (La Co O3) - (La Mn O3) -(Ba Co Oz) (Ba Mn O3) system.
Authors	Cherepanov, V.A.;Filonova, E.A.;Voronin, V.I.;Berger, I.F.
Reference	Journal of Solid State Chemistry (2000) 153, 205-211 Link Xref SCOPUS SCIRUS Google
Compound	La1 Mn1 O3 - Lanthanum manganese trioxide [ABX3] [oP20] [d c2 a] []
Cell	5.4820(9), 7.778(2), 5.5253(9), 90., 90., 90. PNMA (62) V=235.59
Remarks	RVP XDP At least one temperature factor missing in the paper. The coordinates are those given in the paper but the atomic distances do not agree with those calculated during testing.The coordinates are probably correct. No R value given in the paper.

Atom (site)	Oxid.		x, y, z, B, Occupancy			
La1	(4c)	3	0.5184(4)	0.25	0.007(2)	0 1
Mn1	(4a)	3	0	0	0	0 1
O1	(4c)	-2	-.005(7)	0.25	0.075(1)	0 1
O2	(8d)	-2	0.288(9)	0.096(9)	0.23(2)	0 1

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3. Space group symmetry: space group $I4mm$

Symmorphic SG

$I4mm$

No. 107

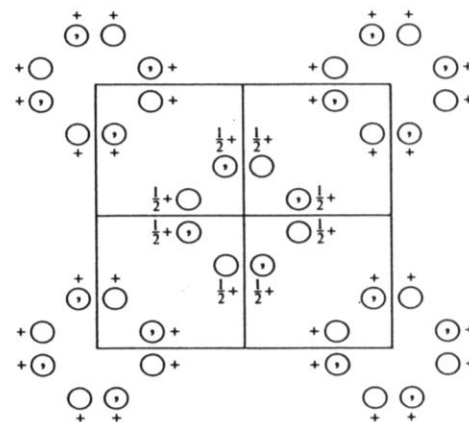
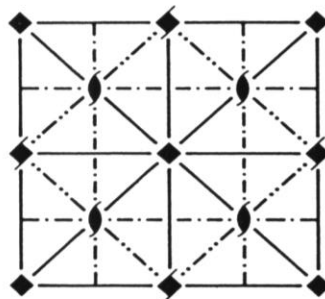
C_{4v}^9

$I4mm$

$4mm$

Tetragonal

Patterson symmetry $I4/mmm$



Origin on $4mm$

Asymmetric unit $0 \leq x \leq \frac{1}{2}; 0 \leq y \leq \frac{1}{2}; 0 \leq z \leq \frac{1}{2}; x \leq y$

Symmetry operations

For $(0,0,0)^+$ set

- | | | | |
|-----------------|-----------------|-----------------------|-------------------|
| (1) 1 | (2) 2 $0,0,z$ | (3) 4^+ $0,0,z$ | (4) 4^- $0,0,z$ |
| (5) m $x,0,z$ | (6) m $0,y,z$ | (7) m x,\bar{x},z | (8) m x,x,z |

For $(\frac{1}{2},\frac{1}{2},\frac{1}{2})^+$ set

- | | | | |
|--|--|--|--|
| (1) $t(\frac{1}{2},\frac{1}{2},\frac{1}{2})$ | (2) $2(0,0,\frac{1}{2})$ $\frac{1}{2},\frac{1}{2},z$ | (3) $4^+(0,0,\frac{1}{2})$ $0,\frac{1}{2},z$ | (4) $4^-(0,0,\frac{1}{2})$ $\frac{1}{2},0,z$ |
| (5) $n(\frac{1}{2},0,\frac{1}{2})$ $x,\frac{1}{2},z$ | (6) $n(0,\frac{1}{2},\frac{1}{2})$ $\frac{1}{2},y,z$ | (7) c $x+\frac{1}{2},\bar{x},z$ | (8) $n(\frac{1}{2},\frac{1}{2},\frac{1}{2})$ x,x,z |

Bravais lattice: body centered (I) tetragonal

Axis 4 $\parallel \vec{c}$; mirrors $m \perp \langle 100 \rangle$; mirrors $m \perp \langle 110 \rangle$

3. Space group symmetry: space group $I4mm$

Symmetry operations

4-fold rotation applied 2, 1, and 3 times, resp.

For $(0,0,0)^+$ set

(1) 1	(2) 2 0,0,z	(3) 4 ⁺ 0,0,z	(4) 4 ⁻ 0,0,z
(5) m x,0,z	(6) m 0,y,z	(7) m x, \bar{x} ,z	(8) m x,x,z

For $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})^+$ set

(1) $t(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	(2) 2(0,0, $\frac{1}{2}$) $\frac{1}{2}, \frac{1}{2}, z$	(3) 4 ⁺ (0,0, $\frac{1}{2}$) 0, $\frac{1}{2}, z$	(4) 4 ⁻ (0,0, $\frac{1}{2}$) $\frac{1}{2}, 0, z$
(5) $n(\frac{1}{2}, 0, \frac{1}{2})$ x, $\frac{1}{2}, z$	(6) $n(0, \frac{1}{2}, \frac{1}{2})$ $\frac{1}{2}, y, z$	(7) c $x + \frac{1}{2}, \bar{x}, z$	(8) $n(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ x,x,z

I lattice

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; $t(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$; (2); (3); (5)

Positions

Multiplicity,
Wyckoff letter,
Site symmetry

Coordinates

(0,0,0)⁺ $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})^+$

I lattice

16	e	1	(1) x,y,z	(2) \bar{x}, \bar{y}, z	(3) \bar{y}, x, z	(4) y, \bar{x}, z
			(5) x, \bar{y}, z	(6) \bar{x}, y, z	(7) \bar{y}, \bar{x}, z	(8) y,x,z

Add (0,0,0) and $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$
to these coordinates
→ 8 × 2 atomic coordinates

8	d	.m.	x,0,z	$\bar{x}, 0, z$	0,x,z	0, \bar{x}, z
8	c	.m	x,x,z	\bar{x}, \bar{x}, z	\bar{x}, x, z	x, \bar{x}, z
4	b	2mm.	0, $\frac{1}{2}, z$	$\frac{1}{2}, 0, z$		
2	a	4mm	0,0,z			

Summary

Point group symmetry:

Allows to **predict** the existence or not of some **macroscopic physical properties**
And in the case they do exist, the direction of the vectorial quantity or form of the tensor, ...

Translation symmetry:

Responsible for diffraction → see lecture III

Structure completely described by:

Space group + lattice parameters + asymmetric unit

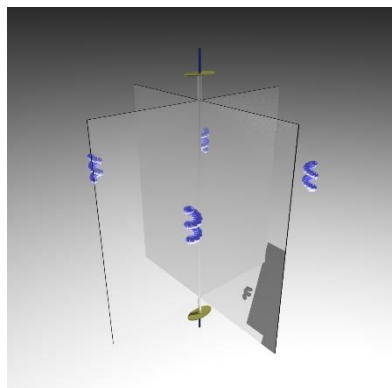
Then use Wyckoff positions to calculate the coordinates of the other atoms of the motif, and last the lattice translations

The same approach can be done for magnetic structures → see lecture II

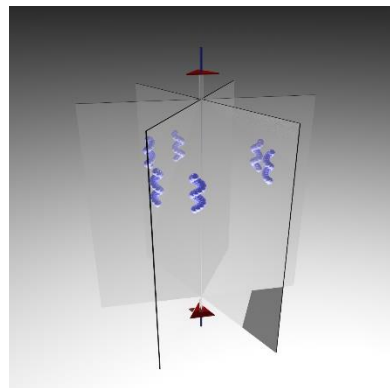
Point group: to go further ...

From crystallographic point groups ...

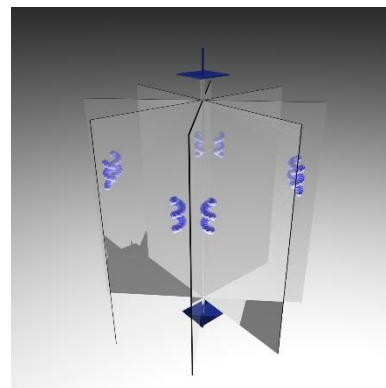
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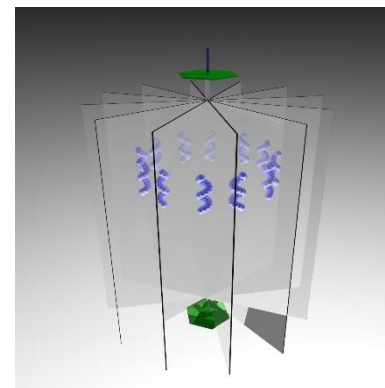
2mm



3m



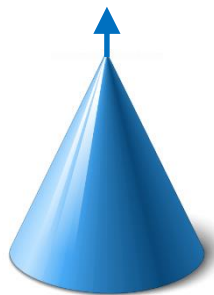
4mm



6mm

... to Curie point groups

∞m



Thank you ... et bonne dégustation

