(High field) Transport properties of strongly correlated metals

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Outline

1. Definitions & Reminders
2. Experimental techniques
3. Transport properties of SCES
4. High field transport measurements
1. Definitions & Reminders
Reminders

Band theory at 2D: \[ \varepsilon(\vec{k}) = -2t\left(\cos(k_x a) + \cos(k_y b)\right) \]

\[ \varepsilon_F \sim 1-10 \text{ eV} \]

Typical excitations (\(\Delta V, \Delta T\)) \(\sim\) meV

Comparison with band structure calculations, effect of interactions, phase transitions...

FS measurements

Global properties: \(C_v, \chi_{\text{Pauli}}, R_H, \Delta \rho/\rho\...\)

Topographic properties: ARPES, AMRO, QO
Reminders

**Global properties**

- **Specific heat**
  
  \[ C_v = \frac{\partial U}{\partial T} = \frac{\pi^2}{3} k_B g(\varepsilon_F) \times T \]

  where

  \[ U = \int_0^{E_F} \varepsilon n(\varepsilon) f(\varepsilon) d\varepsilon \]

  \[ g(\varepsilon_F) = \frac{m^* k_F}{\hbar^2 \pi^2} \]

- **Hall effect**

  \[ R_H = \frac{\rho_{xy}}{B} = \frac{1}{nq} \]

- **Magnetoresistance**

  \[ \bar{J} = ne\bar{v} = e \int_{SF} \bar{v} \frac{\delta k \cdot d\bar{S}}{4\pi^2} \]

  where

  \[ \delta k = \frac{e\tau}{\hbar} \bar{E} \]

\[ \bar{J} = \frac{e^2 \tau}{4\pi^3 \hbar} \int_{SF} \bar{v} \cdot d\bar{S} \bar{E} \]
**Reminders**

**Drude theory**

\[
\sigma = \frac{ne^2\tau}{m^*} \quad [\Omega \text{ cm}]^{-1}
\]

**Electrical conductivity**

\[
\kappa = \frac{1}{3}v_F^2\tau C_v = \frac{1}{3}\ell v_F C_v \quad [\text{W} / \text{K cm}]
\]

**Thermal conductivity**

\[
\frac{\kappa}{\sigma} = \frac{1}{3}m^*v_F^2C_v\frac{ne^2}{ne^2}
\]

Wiedemann-Franz law:

\[
\frac{\kappa}{\sigma} = \frac{1}{3}m^*v_F^2C_v\frac{ne^2}{ne^2}
\]

\[
\text{if} \quad C_v = \frac{3}{2}nk_B \quad \text{and} \quad \frac{1}{2}m^*v_F^2 = \frac{3}{2}nk_B \quad \text{then} \quad \frac{\kappa}{\sigma} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2 T
\]

\[
\lim_{T \to 0} \frac{\kappa}{T\sigma} = L_0 \quad \text{where} \quad L_0 = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2
\]

Universal law, i.e. robust signature of Fermi liquid theory, stating that the electronic carriers of heat are fermionic excitations of charge \(e\).
Reminders

Boltzmann theory

$f_k(r)$ Distribution function which measure the number of carrier $(k, r)$

The distribution function can change through

(i) *Diffusion* Carriers of velocity $v_k$ enter whilst others leave

$$f_k|_{\text{diff}} = -v_k \frac{\partial f_k}{\partial r}$$

(ii) *External fields* $\dot{k} = -\frac{e}{\hbar}(E + v_k \wedge H)$

$$\dot{f}_k|_{\text{field}} = -\frac{e}{\hbar} (E + v_k \wedge H) \frac{\partial f_k}{\partial k}$$

(iii) *Scattering* Several processes throw carries from one state to another through interaction or collision

$$\dot{f}_k|_{\text{scatt}}$$
Total rate of change: \[ \dot{f}_k = \dot{f}_k \bigg|_{\text{diff}} + \dot{f}_k \bigg|_{\text{field}} + \dot{f}_k \bigg|_{\text{scatt}} \]

\[ -\mathbf{v}_k \cdot \frac{\partial f_k}{\partial \mathbf{r}} - \frac{e}{\hbar} (E + \mathbf{v}_k \wedge \mathbf{H}) \cdot \frac{\partial f_k}{\partial \mathbf{k}} = \dot{f}_k \bigg|_{\text{scatt}} \]

and \[ J = \int e \mathbf{v}_k f_k \, dk \]

Boltzmann equation

Rq: (i) Isotropic condition: \[ J = \frac{e^2 \tau}{4\pi^3 \hbar} \int \mathbf{v}_k dS \cdot E \]

(ii) Shockley-Chambers tube integral

\[ \sigma_{\alpha\beta} = -\frac{e^3 B}{2\pi^2 \hbar^2} \int_0^T \left( \int_0^\infty v_x(t) e^{-t'/\tau(t)} v_\beta(t+t') \, dt' \right) \, dt \]
Quantum oscillations

1930  de Haas-van Alphen / Shubnikov-de Haas effect

W.J. de Haas
(1878-1960)

P.M. van Alphen
(1906-1967)

L.V. Shubnikov
(1901-1945)

Bismuth

$10^5 M/H$

$H \perp B$ IN. AXIS.
$\triangle H \parallel B$ IN. AXIS.
$T = 14.2 K$

$H$
Quantum oscillations

\[ E = E_z + E_\perp = \frac{\hbar^2 k_z^2}{2m} + \hbar \omega_c \left( n + \frac{1}{2} \right) \]
\[ \omega_c = \frac{qB}{m_c} \]

\[ n(E) = 2\pi \sqrt{\frac{2m}{\hbar^2}} \sum_{n=0}^{\infty} \frac{1}{\sqrt{E - \hbar \omega_c (n + 0.5)}} \]

Density of states

Oscillation of most electronic properties

Magnetization: de Haas-van Alphen (dHvA)

Resistivity: Shubnikov-de Haas (SdH)
Temperature / Disorder effects on quantum oscillations

\[ \omega_c = \frac{eH}{m^*} \]

\[ k_B T / \Gamma = \frac{\hbar}{\tau} \]

\[ \hbar \omega_c > k_B T \]

• Low T measurements

• Need high quality single crystals

\[ \frac{\hbar \omega_c}{\tau} > \frac{\hbar}{\tau} \Rightarrow \omega_c \tau > 1 \]
Quantum oscillations

**Lifshitz-Kosevich theory (1956)**

\[
\Delta R, \Delta M \propto R_T R_D R_S \sin \left[ 2\pi \left( \frac{F}{B} - \gamma \right) \right]
\]

\[ T \neq 0, \quad p = 1 \]

- Onsager relation \( \Rightarrow A_F \)
- Extremal area

\[
\frac{F}{B} = \frac{\hbar A_F}{2\pi q B}
\]

- \( R_T = \frac{X}{\text{sh}(X)} \) where \( X = 14.694 \times T m_c / B \) \( \Rightarrow m^* \)
- Cyclotron mass

- \( R_D = \exp \left( -\frac{14.694 \times T_D m_c}{B} \right) = \exp \left( -\frac{\pi}{\mu B} \right) \) \( \Rightarrow T_D = \frac{\hbar}{2\pi k_B \tau} \)
- Dingle temperature (mean free path)

- \( R_S = \cos \left( \frac{\pi}{2} m^* g \right) \) \( \Rightarrow m_b^* g \)

**Direct measure of the Fermi surface extremal area**

(but number of orbits? location in k-space?)

**Rq: Luttinger theorem at 2D**

\[
n_{2D} = \frac{2A_k}{(2\pi)^2} = \frac{F}{\phi_0}
\]
Quantum oscillations: the case of $\text{Sr}_2\text{RuO}_4$

$\text{Sr}_2\text{RuO}_4$: a Quasi-2D Fermi liquid

Band structure calculations

3 sheets of FS

$\alpha$ hole like

$\beta$, $\gamma$ electron like

A. Liebsch et al, PRL 84, 1591 (2000)

Quantum oscillations: the case of $\text{Sr}_2\text{RuO}_4$.


<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency $F\ (kT)$</td>
<td>3.05</td>
<td>12.7</td>
<td>18.5</td>
</tr>
<tr>
<td>Average $k_F\ (\text{Å}^{-1})$</td>
<td>0.302</td>
<td>0.621</td>
<td>0.750</td>
</tr>
<tr>
<td>$\Delta k_F/k_F\ (%)$</td>
<td>0.21</td>
<td>1.3</td>
<td>&lt;0.9</td>
</tr>
<tr>
<td>Cyclotron mass ($m_e$)</td>
<td>3.4</td>
<td>6.6</td>
<td>12.0</td>
</tr>
<tr>
<td>Band calc. $F\ (kT)$</td>
<td>3.4</td>
<td>13.4</td>
<td>17.6</td>
</tr>
<tr>
<td>Band calc. $\Delta k_F/k_F\ (%)$</td>
<td>1.3</td>
<td>1.1</td>
<td>0.34</td>
</tr>
<tr>
<td>Band mass ($m_e$)</td>
<td>1.1</td>
<td>2.0</td>
<td>2.9</td>
</tr>
</tbody>
</table>

Figs. 1 and 2: ARPES

2. Experimental techniques
**Electrical transport vs Thermal transport**

- Resistivity
- Hall effect
- Seebeck coefficient
- Nernst coefficient
- Thermal conductivity

**Expressions**

\[
S = \frac{\alpha}{\sigma T} \frac{E_x}{\nabla_x T}
\]

\[
\nu = \frac{N}{B} \frac{1}{B_z} \frac{E_y}{\nabla_x T}
\]

\[
\kappa(T) = \frac{Q}{\Delta T}
\]
Thermal transport setup

Subkelvin temperature measurement:
\[ \kappa(T) = \kappa_{\text{phonon}}(T) + \kappa_{\text{electronic}}(T) \]

- Manganin
- Gold wires
- RuO₂
- Cu

Courtesy of K. Behnia

\[ \kappa/\tau = \begin{cases} 
0 & \text{insulator: } a = 0 \\
0.1 & \text{s-wave superconductor: } a = 0 \\
0.3 & \text{d-wave superconductor: } a \neq 0 \end{cases} \]

YBCO (optimal)
Electrical transport measurements

2 points measurements

4 points measurements

Sample connected with silver paint

Microstructures (FIB carved) of CeRhIn$_5$

~60 x 60 µm

~ 400 µm

Phase-sensitive detection

\[ V_{\text{sig}} = V_{\text{sig}} \sin(\omega_r t + \Theta_{\text{sig}}) \]

\[ V_{\text{ref}} = V_L \sin(\omega_L t + \Theta_{\text{ref}}) \]

Output of a mixer (PSD) = product of two sine waves

\[
0.5 V_{\text{sig}} V_{\text{ref}} \cos([\omega_r - \omega_L]t + \Theta_{\text{sig}} - \Theta_{\text{ref}}) \\
- \quad 0.5 V_{\text{sig}} V_{\text{ref}} \cos([\omega_r + \omega_L]t + \Theta_{\text{sig}} + \Theta_{\text{ref}})
\]

This technique allows to detect the response from the experiment at the reference frequency (narrow band detection much better than a simple filter)

DC signal if \( \Theta_{\text{sig}} - \Theta_{\text{ref}} = \) cte
Lock-in amplifier

SR830 FUNCTIONAL BLOCK DIAGRAM

Reference In Sine or TTL

Low Noise Differential Amp

50/60 Hz Notch Filter

100/120 Hz Notch Filter

Gain

DC Gain Offset Expand

Low Pass Filter

Phase Sensitive Detector

90° Phase Shift

Phase Locked Loop

Internal Oscillator

Phase Shifter

Low Pass Filter

R and Θ Calc

X Out

Y Out

Sine Out

TTL Out

Discriminator

Reference

signal

R

Θ

X

Y
High magnetic field facility

Large high magnetic field facilities (pulsed and DC)
LNCMI-Grenoble: static fields

Resistive coil
\[ I_{\text{max}} = 32\,000\,A,\ P = 24\,\text{MW} \]
Water flow \ ~ 300\ L/s (for cooling)
Max. field = 36.5 T

Hybrid project (2019)
34 T (R) + 9 T (SC) = 43 T

NHMFL Tallahassee (45 T)
LNCMI-Toulouse: pulsed fields
High magnetic fields up 98.8 T

LNCMI-Toulouse: pulsed fields

100 T coil
3. Transport properties of SCES

Inspired by a talk of N. Hussey
Transport properties of SCES

What makes DC transport measurements such an important probe of SCES?

✓ “Often the first thing to be measured, but the last to be understood…”

✓ “What scatters may also pair”

Hence electrical resistivity is a powerful, albeit coarse, probe of superconductivity

✓ In the Fermi liquid picture the resistivity is $T^2$ with $A/\gamma^2 \approx 10^{-5} \, \mu\Omega \, \text{cm} \, \text{mol}^2 \, \text{K}^2/\text{J}^2$

✓ Close to a quantum critical point the resistivity is linear in $T$
High temperature: bad metals
What constitutes metallic behaviour?

**Basic definition:** A material whose resistivity increases with temperature

\[ \rho(T) = \frac{m^*}{ne^2} \Gamma(T) \]

\( \Gamma(T) \sim T^5 \) for electron – phonon

\( \Gamma(T) \sim T^2 \) for electron – electron

\( T \ll \omega_D, E_F \)

\( \omega_D \ll T \ll E_F \)

+ electron – any excitation (magnons...)

\( T \to 0 \)

\[ \rho_0 = \frac{m^*}{ne^2} \Gamma_0 \]
What constitutes metallic behaviour?

**Basic definition:** A material whose resistivity increases with temperature.

According to the Drude model:

\[ \rho(T) = \frac{m^*}{ne^2} \Gamma(T) \propto \frac{1}{\ell(T)} \]

Can a metallic state survive this reduction in \( \ell \) indefinitely?
Ioffe-Mott-Regel limit

\[ \rho(T) = \frac{m^*}{ne^2} \Gamma(T) \propto \frac{1}{\ell(T)} \]

Semiclassical theory breaks down if
\[ \ell \text{ become shorter than the interatomic distance } a \]

OR
\[ \ell > \lambda_F = \frac{2\pi}{k_F} \]

\[ k_F \ell > 1 \]

\[ \ell \approx a \Rightarrow \text{saturation of the resistivity (} \Delta k \sim \text{size of Brillouin zone)} \]

Gunnarson et al, RMP 75 1085 (03)
Conventional metallic transport at high $T$

- Drude term = coherent QP contribution
- Peak centred at $\omega = 0$ but extends up to $W$
- Drude peak broadens at high $T$ with a width at half-maximum equal to $\Gamma(T < T_m)$
- Saturation of $\rho \Leftrightarrow$ Loss of coherence of the QP $\sigma(\omega, T)$ evolves into a plateau ($T < W$)

Saturating metals

$\sigma(\omega) = \frac{\sigma_0}{1 + \omega^2 \tau^2}$

Spectral weight preserved below $\omega \sim W$ (bandwidth)

Hussey et al, Phil. Mag 84 2847 (2004)
**Bad metallic transport in cuprates**

Non saturating metals:
High \( T_c \) cuprates, manganites, vanadates, ruthenates and organics

Universal feature of bad metals:
- Proximity to Mott insulator
- Strong e-e interaction

- Drude peak disappears around \( \sigma_0 \approx 700 – 1000 \, \Omega^{-1}\text{cm}^{-1} \) (value close to the MIR limit)
- Lost spectral weight recovered at \( \omega > W \)

Coherent to incoherent cross-over in the optical conductivity along with the loss of spectral weight (within \( W \)) may explain the non saturating resistivity in ‘bad metals’
Conventional vs bad metallic transport at high $T$

A bad metal behaves as if it is a QP insulator which is render metallic by collective fluctuations (e.g. CDW, SDW, stripes...)

- Low $\omega$ dip
- $T$-indept $\sigma(\omega)$
- at high $\omega$
- $T_{\text{crit}}$ and $\sigma_{\text{crit}}(0)$
Low temperature: $T^2$ resistivity
Correlated Fermi liquid at low $T$

$T^2$ resistivity originates from electron-electron scattering processes near $E_F$

Electrons participating in the scattering event are those confined to a width of $k_B T / E_F$

Overdoped cuprates


Heavy fermions

Lohneysen, JPCM 8 9689 (1996)
Kadowaki-Woods ratio

\[ \rho(T) = \rho_0 + AT^2 \]
\[ C_{el} = \gamma T = \frac{\pi^2}{6} k_B^2 N(\varepsilon_F)T \]
\[ A/\gamma^2 = \text{const.} \rightarrow A \propto N(\varepsilon_F)^2 \]

Yamada & Yoshida, Prog.Theor.Phys. 76 621 (86)
Auerbach & Levin, JAP 61 3162 (87)
Coleman, PRL 59 1026 (87)
Miyake, Matsuura & Varma, SSC 71 1149 (89)
Kontani, JPSJ 73 515 (04)

Kadowaki & Woods, SSC 58 507 (86)

\[ A/\gamma^2 \sim a_0 = 10^{-5} \mu\Omega \text{cm.mol}^2 . K^2 / J^2 \]

\[ A \propto \gamma^2 \propto m^*^2 \]

\[ A_i = \left( \frac{8\pi^3 a c k_B^2}{e^2 \hbar^3} \right) \left( \frac{m_i^2}{k_{Fi}} \right) \]

Hussey, JPSJ 74 1107 (2005)
Correlated Fermi liquid at low $T$

\[ \rho = \rho_0 + A T^2 \]

\[ A_i = \left( \frac{8\pi^3 a c k_i^2}{e^2 \hbar^3} \right) \left( \frac{m_i^*}{k_i^3} \right) \]

**Tl$_2$Ba$_2$CuO$_{6+\delta}$**

- $A = 5.4 \pm 0.5 \text{ n}\Omega \text{ cm K}^{-2}$
- $A_{KWR} = 3.9 \text{ n}\Omega \text{ cm K}^{-2}$

**Sr$_2$RuO$_4$**

- $A = 5 \pm 1.5 \text{ n}\Omega \text{ cm K}^{-2}$
- $A_{KWR} = 3.6 \text{ n}\Omega \text{ cm K}^{-2}$

- Multi-band system $\Rightarrow \frac{1}{A} = \sum_i \frac{1}{A_i}$

- **C. Proust et al, PRL'01**
- **N.E. Hussey et al, PRB'98**
- **B. Vignolle et al, Nature'08**
- **C. Bergemann et al, Ad. Phys.'03**
Low temperature: Quantum criticality
Quantum criticality

‘Classical’ phase transition

Melting line

Phase transition that is driven not by T but by quantum fluctuations (‘zero point motion’)

\[ \Delta x \cdot \Delta p \geq \hbar \]

As \( T \to 0 \), thermal motion ceases but electron cannot be at rest (\( \Delta x \) and \( \Delta p \) fixed)

\[ \Rightarrow \text{« State of constant agitation »} \]

that can melt order
Quantum critical metals

\[ \rho = \rho_0 + A' T^\phi \]

\[ \rho \sim T \]

\[ C_v \sim \ln \frac{1}{T} \]

\[ C_v \sim m^* \Rightarrow m^* \to \infty \text{ at a QCP!} \]

Lohneysen, *JPCM* 8, 9689 (1996)

Quantum critical metals

Organic superconductors
Doiron-Leyraud et al, PRB 80 214531 (2009)

Pnictides
Shibauchi et al, ARCMP 5 1113 (2014)

Cuprates
Origin of the T-linear resistivity

\[ \rho \sim T \]

Competition of weak, but isotropic impurity scattering and strong scattering from spin-fluctuations

\[ \rho \sim T^2 \]

Proposition: scattering near a QCP have a local character, i.e. no k-dependence
⇒ the entire FS is "hot“ : Marginal Fermi liquid with \( \rho \sim T \)
Similar scattering rate per kelvin of metals in which resistivity is linear in $T$ (QCP or e-ph scattering)

For SCES, $0.9 < \alpha < 2.2$ in spite of differences in dimensionality and microscopic nature of the interactions.

The law of quantum mechanism forbids the dissipation time to be any shorter then $\tau$.
4. High fields transport properties
Why high magnetic field?

Resolve Fermiology

\[
\text{BaFe}_2(\text{As}_1-x\text{P}_x)_2
\]

Induce New States of Matter & New Properties

Restore the Normal State of Superconductors

H. Shishido et al. PRL 104, 057008 (2010)


« Although it is difficult to predict the role that quantum criticality will play in our final understanding of the cuprates, the case for a QCP would be made very compelling if a new experiment were to reveal a sharp and pronounced change in some electronic property in the zero-temperature limit, on crossing the QCP as a function of doping. »

Pseudogap and quantum criticality in cuprates

Partial suppression of the low energy excitation as seen by spectroscopy and thermodynamic probes and located at the anti-node

Doped Mott insulator?

Phase with a distinct order parameter?

\[ K \propto \chi'(q=0, \omega) \propto \text{DOS} \]

Curro et al, PRB 56, 877 (1997)
Pseudogap and quantum criticality in cuprates

Pseudogap = partial suppression of the low energy excitation as seen by spectroscopy and thermodynamic probes and located at the anti-node (from ARPES)

The broken symmetries are instability of the pseudogap
Pseudogap and quantum criticality in cuprates

Low T (H = 80 T)

- Pseudogap and charge order are separate phenomena.
- Sharp drop of the carrier density at the critical point of the pseudogap.

S. Badoux et al., Nature (2016)
High fields transport properties in LSCO

Access ground state using pulsed magnetic fields

$T$-linear term becomes dominant at low $T$

Cooper et al., Science 323, 603 (2009)
Anomalous criticality in LSCO

$\Delta \rho_{ab}(T) = \alpha_n T^n$

$\rho_{ab}(T)$ remains $T$-linear at low $T$ over an anomalously wide doping range.


La$_{2-x}$Sr$_x$CuO$_4$
Anomalous criticality in LSCO

Cooper et al., Science 323, 603 (2009)

$\rho_{ab}(T)$ remains $T$-linear at low $T$ over an anomalously wide doping range

$\Delta \rho_{ab}(T') = \alpha_n T^n$

$\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$
Doping dependence of the linear term of the resistivity in hole-doped cuprates

“What scatters may also pair”

Taillefer, *ARCMP* 1, 51 (2010)